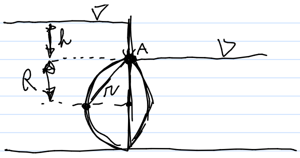
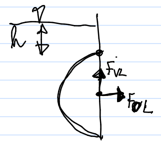
Correzione Esame di Biofluidodinamica del 06/09/2018

Esercizio n. 1



? h PER EQUILIBRIO

LEFT



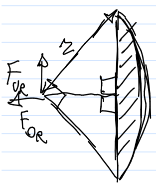
$$F_{VL} = \frac{\pi R^2}{2} b \rho g \quad \text{BRACCIO NULLO}$$

$$M_{F_{VL}} = 0$$

$$F_{OL} = \rho g (h+R) 2Rb$$

$$M_{F_{OL}} = \rho g (h+R) 2R^2 b$$

RIGHT $\quad r = \sqrt{R^2+R^2} = \sqrt{2}$



$$Vol = \frac{\pi r^2}{4} b - \frac{R^2}{2} b$$

$$= \frac{\pi 2R^2}{4} b \quad R^2 b$$

$$= \left(\frac{\pi}{2} - 1\right) R^2 b$$

$$F_{VR} = \rho g \left(\frac{\pi}{2} - 1\right) R^2 b$$

$$M_{F_{VR}} = -\rho g \left(\frac{\pi}{2} - 1\right) R^3 b$$

$$F_{OR} = \rho g R \, 2Rb = 2 \rho g R^2 b$$

$$M_{F_{OR}} = -2 \rho g R^3 b$$

$$M_R = -\rho g \frac{\pi}{2} R^3 b + \rho g R^3 b - 2 \rho g R^3 b$$

$$= -\rho g \left(\frac{\pi}{2} + 1\right) R^3 b$$

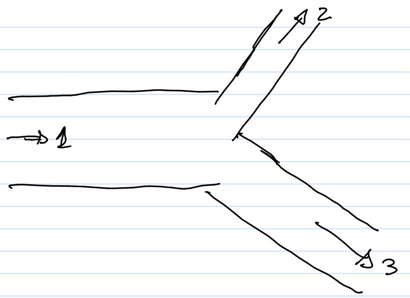
$$M_L = \rho g (h+R) 2R^2 b$$

$$M_L = M_R \Rightarrow 2(h+R) = \left(\frac{\pi}{2} + 1\right) R$$

$$2h = \frac{\pi}{2} R + R - 2R$$

$$= \left(\frac{\pi}{2} - 1\right) \frac{R}{2}$$

Esercizio n. 2



$Q_1 = 0{,}15\ m^3/s$ $\qquad Q_2 = Q_3 = \frac{1}{2} Q_1$

$P_1 = 170\ kPa$

$D_1 = 0{,}15\ m$

$D_2 = 0{,}075\ m$

$D_3 = 0{,}1\ m$

$$U_1 = \frac{Q_1}{\pi D_1^2/4} = 8{,}49\ m/s$$

$$U_2 = \frac{2 Q_1}{\pi D_2^2} = 16{,}98\ m/s$$

$$U_3 = \frac{2 Q_1}{\pi D_3^2} = 9{,}55\ m/s$$

BERNOULLI

$$\frac{p}{\rho} + \frac{U^2}{2} + g z = \text{COSTANTE}$$

$$P_2 = P_1 + \frac{\rho}{2}\left(U_1^2 - U_2^2\right) = 61880\ Pa$$

$$P_3 = P_1 + \frac{\rho}{2}\left(U_1^2 - U_3^2\right) = 160440\ Pa$$

CONS QDM x

$$-\rho U_1^2 A_1 + \rho U_2\, U_2 \cdot \vec{i} A_2 + \rho U_3\, U_3 \cdot \vec{i}\ _3 =$$
$$P_1 A_1 - P_2 A_2\, \vec{n} \cdot \vec{i} - P_3 A_3\, \vec{n} \cdot \vec{i} + F_x$$

$$F_x = -\rho U_1^2 A_1 + \rho U_2^2 A_2 \cos 60^\circ + \rho U_3^2 A_3 \cos 45^\circ$$
$$- P_1 A_1 + P_2 A_2 \cos 60^\circ + P_3 A_3 \cos 45^\circ$$
$$= -\left(\rho U_1^2 + P_1\right) A_1 + \left(\rho U_2^2 + P_2\right) A_2 \cos 60^\circ +$$
$$+ \left(\rho U_3^3 + P_3\right) A_3 \cos 45^\circ$$
$$= -2107\ N$$

Esercizio n. 3

1. Noto
$$\omega = f\left(D, H, \rho, \mu, V\right)$$
ho $n = 6$ variabili e $m = 3$ dimensioni fondamentali coinvolte:
$$[\omega] = T^{-1}, \quad [D] = L, \quad [H] = L, \quad [\rho] = ML^{-3}, \quad [\mu] = ML^{-1}T^{-1}, \quad [V] = LT^{-1},$$
Stimo come $j = m$ la riduzione $j = n - d$. Mi attendo quindi $d = n - j = n - m = 6 - 3 = 3$ gruppi adimensionali, cioè Π_i, Π_2 e Π_3. Una possibile scelta per le variabili ripetute è D,V e μ, ottenendo i gruppi:
$$\Pi_1 = \frac{H}{D},$$
$$\Pi_2 = \frac{\rho V D}{\mu} \qquad \text{(Numero di Reynolds)},$$
$$\Pi_3 = \frac{\omega D}{V} \qquad \text{(Numero di Strouhal)}.$$

2. Nell'ipotesi di similitudine completa posso calcolare la dimensione H_m del modello
$$H_m = \frac{D_m}{D} H = \frac{0.02m}{0.1m} 0.3m = 0.06m$$
e la velocità del flusso intorno al modello
$$V_m = \frac{\rho}{\rho_m} \frac{\mu_m}{\mu} \frac{D}{D_m} V = \frac{1.23kg/m^3}{998kg/m^3} \frac{1 \times 10^{-3}kg/(m \cdot s)}{1.79 \times 10^{-5}kg/(m \cdot s)} \frac{0.1m}{0.02m} 13.9m/s = 4.79m/s$$

3. La frequenza del rilascio dei vortici è quindi pari a
$$\omega = \frac{V}{V_m} \frac{D_m}{D} \omega_m = \frac{13.9m/s}{4.79m/s} \frac{0.02m}{0.1m} 49.4Hz = 28.7Hz$$