ECONOMIA DEGLI INTERMEDIARI FINANZIARI AVANZATA

MODULO ASSET MANAGEMENT

LECTURE 6

MVO IN TWO STAGES

- Calculate the forecasts

  - Calculate forecasts for returns, standard deviations and correlations for the set of assets in which you can invest
    - This is often done using historical data.

- Calculate the Efficient Frontier.
  - The efficient frontier is the set of portfolios that minimizes risk at the possible levels of return.

- A portfolio can be selected from the frontier based on risk, utility maximization, maximum Sharpe Ratio, etc.

- MVO OPTIMIZATION STEP

- Create or calculate Forecasts for Return, Risk and Correlations for a set of assets. These parameters describe a multivariate return distribution

- Calculate the Efficient Frontier.
  - Assume that all portfolios have positive weights (no short-selling) and add to 100.
  - Calculate the minimum variance portfolios and maximum return portfolio using the forecasts.
  - Calculate the portfolio that minimizes risk for each of 98 portfolios between the minimum variance and maximum return portfolios. This set of 100 portfolios is the efficient frontier

MVO LIMITS (& SOLUTIONS)

- Returns are very difficult to forecast.
  - MVO requires forecasts on ALL assets.
  - Historical returns are very poor forecasts.

- Input Sensitivity--MVO is highly sensitive to the return forecasts.
  - Small changes in return assumptions often lead to large changes in the optimal allocations.
  - Estimation Error is built into forecasting and magnified by MVO

- Portfolios are very concentrated (no diversification).

- Portfolios are unintuitive.

- Both of these issues must be solved to make MVO a practical real-world tool.

- **Black-Litterman**
  - Technique developed by Fischer Black and Robert Litterman of Goldman Sachs to create better return estimates.

- **Resampling**
  - Technique developed by Richard Michaud to average over the statistical equivalence region and create a new efficient frontier.

IMPLIED RETURNS

- The Black-Litterman Model starts with Implied Returns
- Other Names for Implied Returns:
  - CAPM Returns
  - Reverse Optimized Returns
  - Market Returns
  - Consensus Returns
  - Equilibrium Returns
- The return of any asset or asset class can be separated into three parts:
  - Risk-Free Return
  - Return Correlated with Benchmark
  - Return Not Correlated with Benchmark
- Returns that are correlated with the benchmark result in beta risk (systematic risk, benchmark risk, non-diversifiable risk, or market risk)
- Beta risk is the type of risk that is rewarded with a premium

IMPLIED RETURNS USING CAPM

- Expected returns are a function of beta risk

$$\underline{E(R_i)} = \underline{R_f} + \underline{\beta_{i,B}}\underline{(R_B - R_f)}$$

R_f	Risk-Free Rate
$\beta_{i,b}$	Beta of Asset Class i Relative to Benchmark
R_b	Return of Benchmark
$R_b - R_f$	Forward Looking Risk Premium of Benchmark = Return of Benchmark over the Risk-Free

In this case the benchmark is the market capitalization weighted portfolio.

IMPLIED RETURNS: RISK AVERSION COEFFICIENT

- The Risk Aversion Coefficient characterizes the risk-return trade-off.
- Using historical risk premium and variance, we got a λ of approximately **3.37**

$$\lambda = \frac{R_B - r_f}{\sigma_B^2} = \frac{\text{Risk Premium}}{\text{Variance}}$$

σ_B^2 = Variance of the Benchmark

IMPLIED RETURNS USING REVERSE OPTIMIZATION

- The same excess returns results from reverse optimization
- The denominator is basically the variance of market portfolio. The numerator is the covariance of the assets in the market portfolio. Asset weights are the equilibrium weights. Covariance matrix ∑ is historical covariance

$$\Pi = \lambda \Sigma w_{mkt}$$

Π = Implied Excess returns over the risk free rate (*N x 1*)

λ = Risk aversion coefficient (*1 x 1*)

Σ = Covariance matrix of returns (*N x N*)

w_{mkt} = Market capitalization weight of the assets (*N x 1*)

EXAMPLE

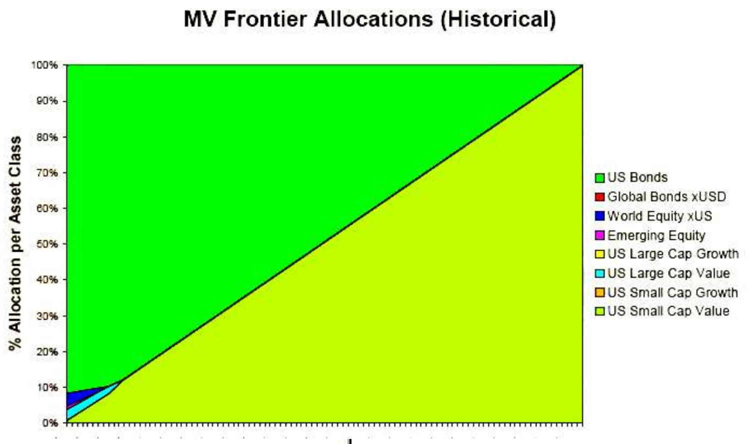
$$\Pi = \lambda \Sigma w_{mkt}$$

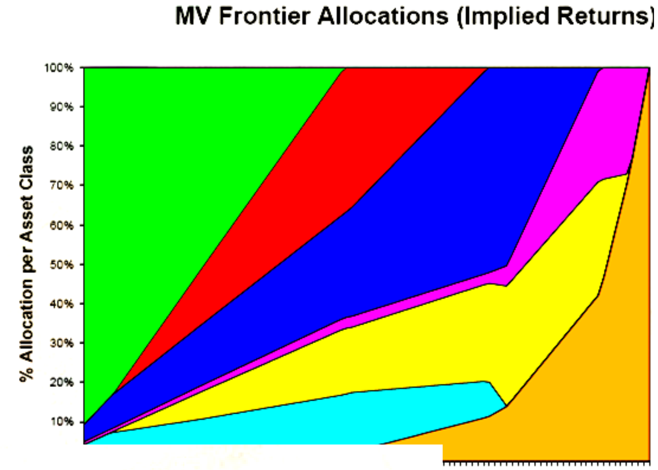
$$\begin{bmatrix} 0.08\% \\ 0.95\% \\ 3.95\% \\ 5.37\% \\ 5.14\% \\ 3.68\% \\ 6.12\% \\ 3.50\% \end{bmatrix} = 3.37 \begin{bmatrix} .0014 & .0015 & -.0008 & -.0017 & -.0010 & -.0007 & -.0015 & -.0006 \\ .0015 & .0076 & .0026 & -.0006 & -.0013 & -.0003 & -.0002 & .0005 \\ -.0008 & .0026 & .0251 & .0292 & .0208 & .0147 & .0248 & .0134 \\ -.0017 & -.0006 & .0292 & .0663 & .0359 & .0244 & .0490 & .0268 \\ -.0010 & -.0013 & .0208 & .0359 & .0468 & .0283 & .0520 & .0260 \\ -.0007 & -.0003 & .0147 & .0244 & .0283 & .0252 & .0314 & .0215 \\ -.0015 & -.0002 & .0248 & .0490 & .0520 & .0314 & .0809 & .0411 \\ -.0006 & .0005 & .0134 & .0268 & .0260 & .0215 & .0411 & .0276 \end{bmatrix} \begin{bmatrix} 20.16\% \\ 27.93\% \\ 22.21\% \\ 2.33\% \\ 12.58\% \\ 12.58\% \\ 1.11\% \\ 1.11\% \end{bmatrix}$$

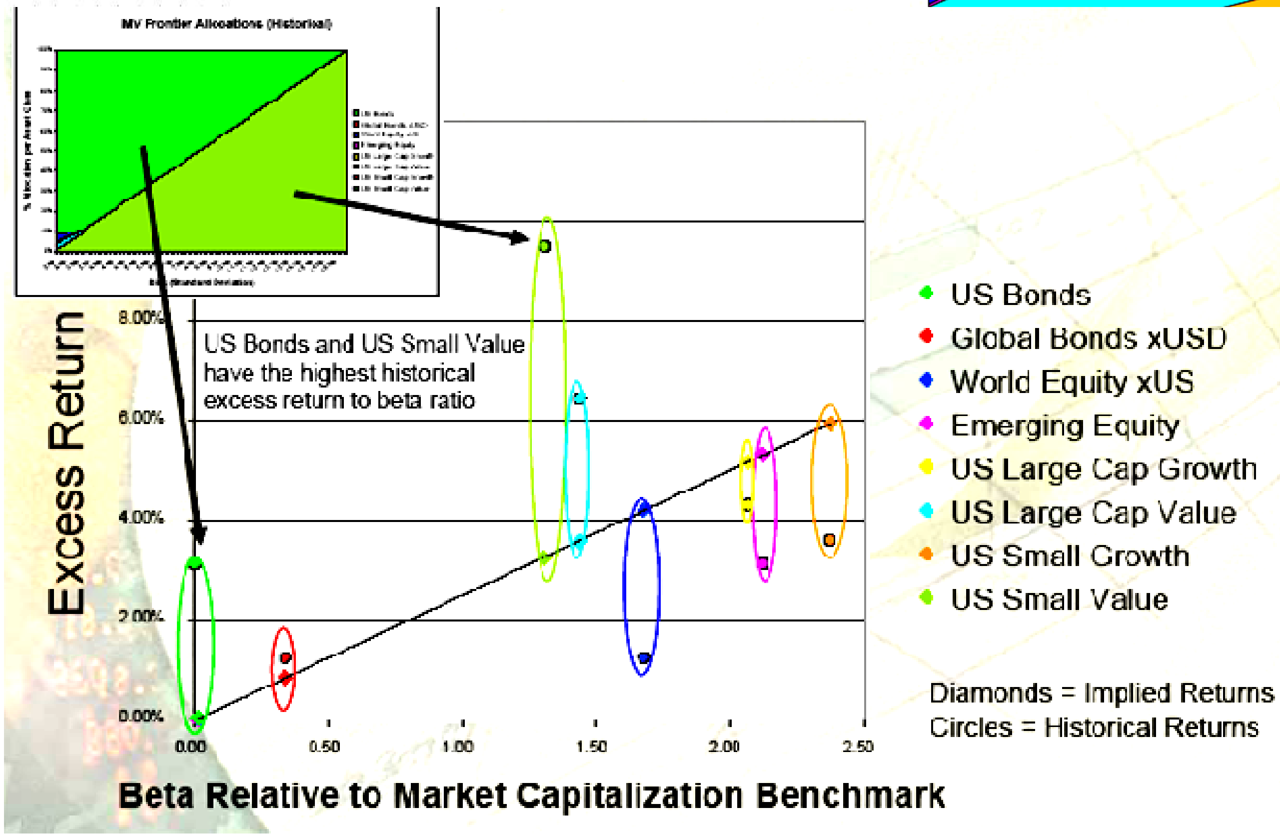
Asset Class	Market Capitalization Estimate	Market Capitalization Weights w_{mkt}
US Bonds	$8,360,741,000,000	20.16%
Global Bonds xUSD	$11,583,275,710,000	27.93%
World Equity xUS	$9,212,460,000,000	22.21%
Emerging Equity	$964,647,000,000	2.33%
US Large Cap Growth	$5,217,844,438,500	12.58%
US Large Cap Value	$5,217,844,438,500	12.58%
US Small Cap Growth	$459,897,061,500	1.11%
US Small Cap Value	$459,897,061,500	1.11%
Total	$41,476,606,710,000	100.00%

Asset Class	Implied Excess Return		Risk-Free Rate		Total Implied Return
US Bonds	0.08%	+	4.00%	=	4.08%
Global Bonds xUSD	0.95%	+	4.00%	=	4.95%
World Equity xUS	3.95%	+	4.00%	=	7.95%
Emerging Equity	5.37%	+	4.00%	=	9.37%
US Large Cap Growth	5.13%	+	4.00%	=	9.13%
US Large Cap Value	3.68%	+	4.00%	=	7.68%
US Small Cap Growth	6.12%	+	4.00%	=	10.12%
US Small Cap Value	3.50%	+	4.00%	=	7.50%

HISTORICAL VS IMPLIED RETURNS OPTIMIZATION







BLACK- LITTERMAN MODEL

- Start with the market returns using reverse optimization and CAPM.

- You may or may not agree with the implied returns

- If you don't agree with the implied returns, the Black-Litterman model provides an elegant framework for combining the implied returns with your unique views that results in well-diversified portfolios that reflect your views

- B-L model uses a Bayesian approach to combine the subjective views of an investor regarding the expected returns of one or more assets with the market equilibrium vector of expected returns (the prior distribution) to form a new mixed estimate of expected returns (the posterior distribution).

- Apply your own unique views of how certain markets are going to behave.
- The end result includes both a set of expected returns of assets as well as the optimal portfolio weights.
- If you do not have views, you hold the market portfolio (the benchmark).

- Your views will tilt the final weights away from the market portfolio, the degree to which depending on how confident you are about your views.

- Types of view:
  - Absolute Views — Asset A will have a return of X%
  - Relative Views — Asset A will outperform Asset B by X%

FORMING VIEW

- Our view is: $Q=Pu+\eta$, $\mu\sim\Phi(0,\Omega)$
- Note: same as $Pu=Q+\eta$, because $\eta\sim\Phi(0,\Omega) \Leftrightarrow -\eta\sim\Phi(0,\Omega)$
- u is the expected future returns (a NX1 vector of random variables).
- Ω is assumed to be diagonal

- What does this $Q=P*u+\eta$, Or equivalently $P*u=Q+\eta$ mean?
- Look at P*u:

  each row of P represents a set of weights on the N assets, in other words, each row is a portfolio of the N assets. (aka “view portfolio”)

  u is the expected return vector of the N assets
- P*u means we are expressing our views through k view portfolios

- Expressing views so important because the practical value of BL model lies in the View Expressing Scheme; the model itself is just a publicly available view combining engine.
  - ❖ Our view is the source of alpha.
  - ❖ Expressing views quantitatively means efficiently and effectively translate fundamental analyses into Views

THE NEXT STEP IS TO DO MARKOWITZ MVO

- By using the combined forecasted means and the forecasted covariance matrix ∑.
- So we start with Markowitz (reverse optimization) and CAPM (implied beta).
- Go though Black-Litterman View Combining engine.
- And end up with Markowitz again with predictive means, (and forward looking return covariance matrix.)

- First bracket "[ ]" (role of "Denominator") : Normalisation
- Second bracket "[ ]" (role of "Numerator") : Balance between returns Π (equilibrium returns) and Q (Views). Covariance $(\tau\ \Sigma)^{-1}$ and confidence $P'\ \Omega^{-1}P$ serve as weighting factors, and

$$P'\ \Omega^{-1}Q = P'\ \Omega^{-1}P\ P^{-1}\ Q$$

- Extreme case 1: no estimates P=0: $E(R)$ = Π i.e. BL-returns = equilibrium returns.
- Extreme case 2: no estimation errors $\Omega^{-1} \rightarrow \infty$: $E(R) = P^{-1}Q$ i.e. BL-returns = View returns.

$$E[R] = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right]$$

$E[R]$ = New Combined Return Vector (N x 1 column vector)

τ = Scalar

Σ = Covariance Matrix (N x N matrix);

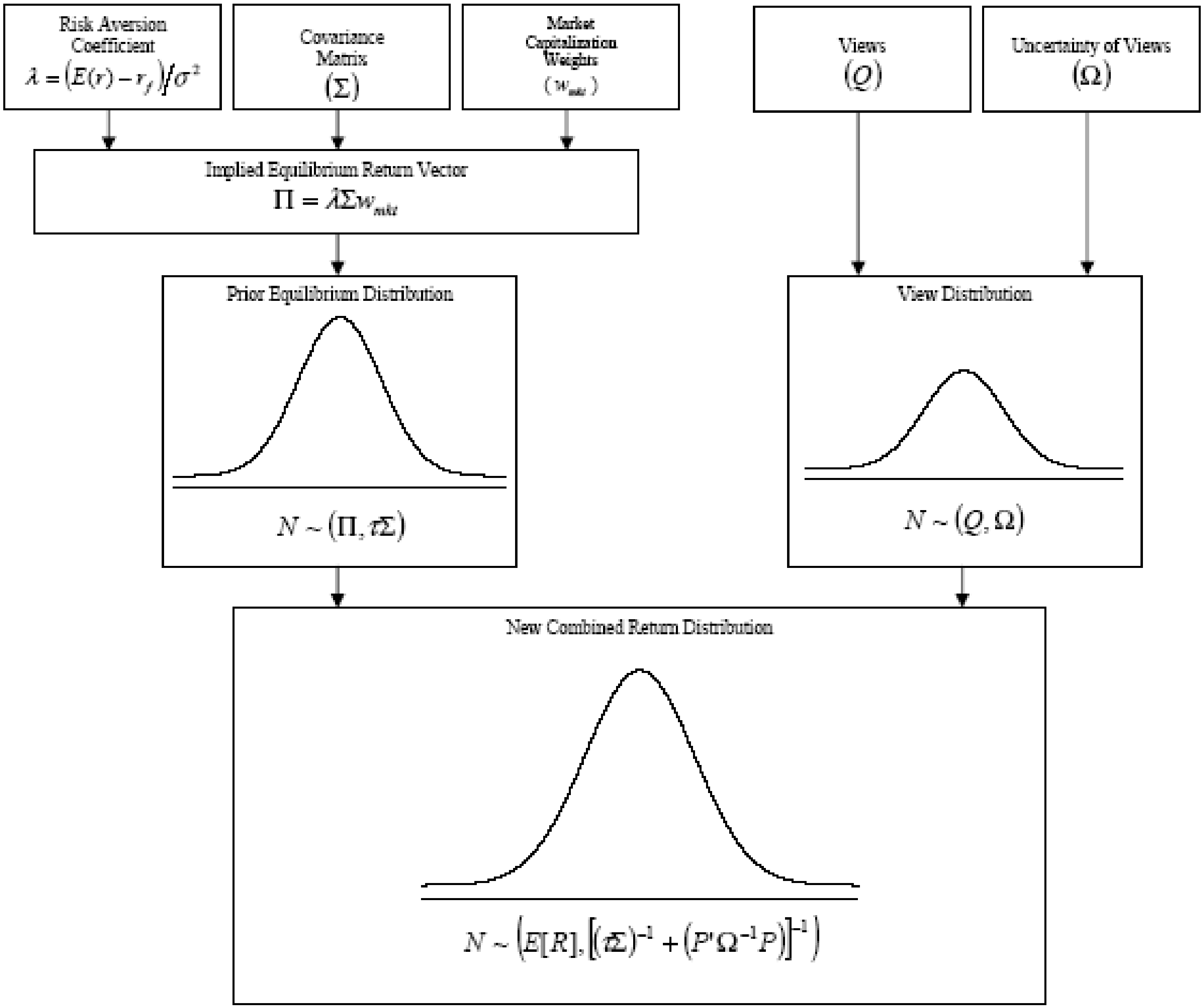
P = View Participation Matrix (K x N matrix that identifies the assets involved in the views)

Ω = Diagonal covariance matrix of error terms from the expressed views representing the uncertainty in each view (K x K matrix);

Π = Implied Excess returns over the risk free rate (N x 1 column vector)

Q = View Vector (K x 1 column vector)

ROAD MAP


Risk Aversion Coefficient
$\lambda = (E(r) - r_f)/\sigma^2$
Covariance Matrix
(Σ)
Market Capitalization Weights
(w_{mkt})
Views
(Q)
Uncertainty of Views
(Ω)
Implied Equilibrium Return Vector
$\Pi = \lambda \Sigma w_{mkt}$
Prior Equilibrium Distribution
$N \sim (\Pi, \tau\Sigma)$
View Distribution
$N \sim (Q, \Omega)$
New Combined Return Distribution
$N \sim \left(E[R], \left[(\tau\Sigma)^{-1} + (P'\Omega^{-1}P)\right]^{-1}\right)$

AN EIGHT ASSETS EXAMPLE…

- μ_{Hist} is historical mean asset returns
- μ_p is calculated relative to the market cap. weighted portfolio using implied betas and CAPM model.
- Market portfolio weights w_{mkt} is based on market capitalization for each of the assets

Asset Class	μ_{Hist}	μ_P	w_{mkt}
US Bonds	3.15%	0.08%	19.34%
Int'l Bonds	1.75%	0.67%	26.13%
US Large Growth	-6.39%	6.41%	12.09%
US Large Value	-2.86%	4.08%	12.09%
US Small Growth	-6.75%	7.43%	1.34%
US Small Value	-0.54%	3.70%	1.34%
Int'l Dev. Equity	-6.75%	4.80%	24.18%
Int'l Emerg. Equity	-5.26%	6.60%	3.49%
Weighted Average	-1.97%	3.00%	
Standard Deviation	3.73%	2.53%	
High	3.15%	7.43%	26.13%
Low	-6.75%	0.08%	1.34%

COVARIANCE MATRIX ∑

Asset Class	1	2	3	4	5	6	7	8
1. US Bonds	0.001005	0.001328	-0.000579	-0.000675	0.000121	0.000128	-0.000445	-0.000437
2. Intl Bonds	0.001328	0.007277	-0.001307	-0.00061	-0.002237	-0.000989	0.001442	-0.001535
3. US Large Growth	-0.000579	-0.001307	0.059852	0.027588	0.063497	0.023036	0.032967	0.048039
4. US Large Value	-0.000675	-0.000610	0.027588	0.029609	0.026572	0.021465	0.020697	0.029854
5. US Small Growth	0.000121	-0.002237	0.063497	0.026572	0.102488	0.042744	0.039943	0.065994
6. US Small Value	0.000128	-0.000989	0.023036	0.021465	0.042744	0.032056	0.019881	0.032235
7. Int'l Dev. Equity	-0.000445	0.001442	0.032967	0.020697	0.039943	0.019881	0.028355	0.035064
8. Int'l Emerg.Equity	-0.000437	-0.001535	0.048039	0.029854	0.065994	0.032235	0.035064	0.079958

IMPLIED MARKET RETURNS Π(NX1)

$$\Pi = \lambda \Sigma w_{mkt}$$

Asset Class	μ_{Hist}	μ_{P}	Π
US Bonds	3.15%	0.08%	
Int’l Bonds	1.75%	0.67%	
US Large Growth	-6.39%	6.41%	
US Large Value	-2.86%	4.08%	
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Int’l Emerg. Equity	-5.26%	6.60%	
Weighted Average	-1.97%	3.00%	
Standard Deviation	3.73%	2.53%	
High	3.15%	7.43%	
Low	-6.75%	0.08%	

WHAT IS A VIEW?

- Opinion: International Developed Equity will be doing well

- Absolute view:
  - View 1: International Developed Equity will have an absolute excess return of 5.25% (Confidence of view = 25%)

- Relative view:
  - View 2: International Bonds will outperform US bonds by 25 bp (Confidence of view = 50%)
  - View 3: US Large Growth and US Small Growth will outperform US Large Value and US Small Value by 2% (Confidence of View = 65%)

WHAT IS THE VIEW VECTOR Q LIKE?

- Unless a clairvoyant investor is 100% confident in the views, the error term ε is a positive or negative value other than 0
- The error term vector does not enter the Black – Litterman formula; instead, the variance of each error term (ω) does.

$$Q + \varepsilon = \begin{pmatrix} 5.25\% \\ 0.25\% \\ 2.00\% \end{pmatrix} + \begin{pmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \varepsilon_3 \end{pmatrix}$$

WHAT IS THE VIEW MATRIX P LIKE?

- View 1 is represented by row 1. The absolute view results in the sum of row equal to 1
- View 2 & 3 are represented by row 2 & 3. Relative views results in the sum of rows equal to 0
- The weights in view 3 are based on relative market cap. weights, with outperforming assets receiving positive weights and underperforming assets receiving negative weights

P =

	US Bonds	Intl Bonds	US Lg Growth	US Lg Value	US Sml Growth	US Sml Value	Int'l Dev. Eqt	Int'l Emerg.Eqt
Row 1	0	0	0	0	0	0	1	0
Row 2	-1	1	0	0	0	0	0	0
Row 3	0	0	0.9	-0.9	0.1	-0.1	0	0

FINALLY, THE COVARIANCE MATRIX OF THE ERROR TERM Ω

- Ω is a diagonal covariance matrix with 0's in all of the off-diagonal positions, because the model assumes that the views are independent of each other
- This essentially makes ω the variance (uncertainty) of views

$$\boldsymbol{\Omega} = \begin{pmatrix} 0.0007089 & 0 & 0 \\ 0 & 0.000141 & 0 \\ 0 & 0 & 0.000866 \end{pmatrix}$$

RETURN VECTOR & RESULTING PORTFOLIO WEIGHTS

Asset Class	E[R]	Π	E[R]-Π	$\hat{w}$	$\hat{w}_{norm}$	w_{mkt}	$\hat{w} - w_{mkt}$
US Bonds	0.07%	0.08%	-0.02%	29.88%	28.83%	19.34%	10.54%
Int'l Bonds	0.50%	0.67%	-0.17%	15.59%	15.04%	26.13%	-10.54%
US Large Growth	6.50%	6.41%	0.08%	9.35%	9.02%	12.09%	-2.73%
US Large Value	4.32%	4.08%	0.24%	14.82%	14.30%	12.09%	2.73%
US Small Growth	7.59%	7.43%	0.16%	1.04%	1.00%	1.34%	-0.30%
US Small Value	3.94%	3.70%	0.23%	1.65%	1.59%	1.34%	0.30%
Int'l Dev. Equity	4.93%	4.80%	0.13%	27.81%	26.84%	24.18%	3.63%
Int'l Emerg. Equity	6.84%	6.60%	0.24%	3.49%	3.37%	3.49%	0.00%
			Sum	103.63%	100%	100%	3.63%

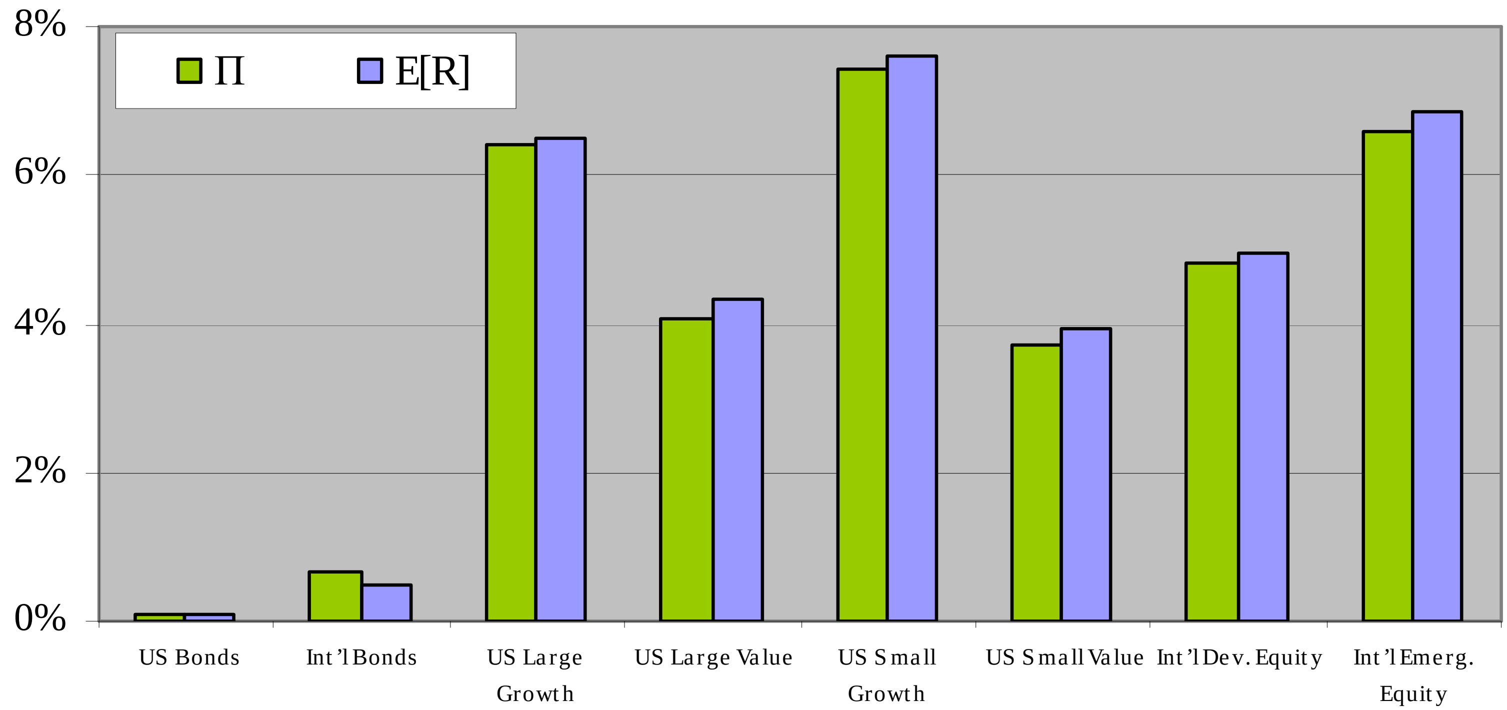
$$\Pi = \lambda \Sigma w_{mkt}$$

$$w = (\lambda\Sigma)^{-1}\Pi$$

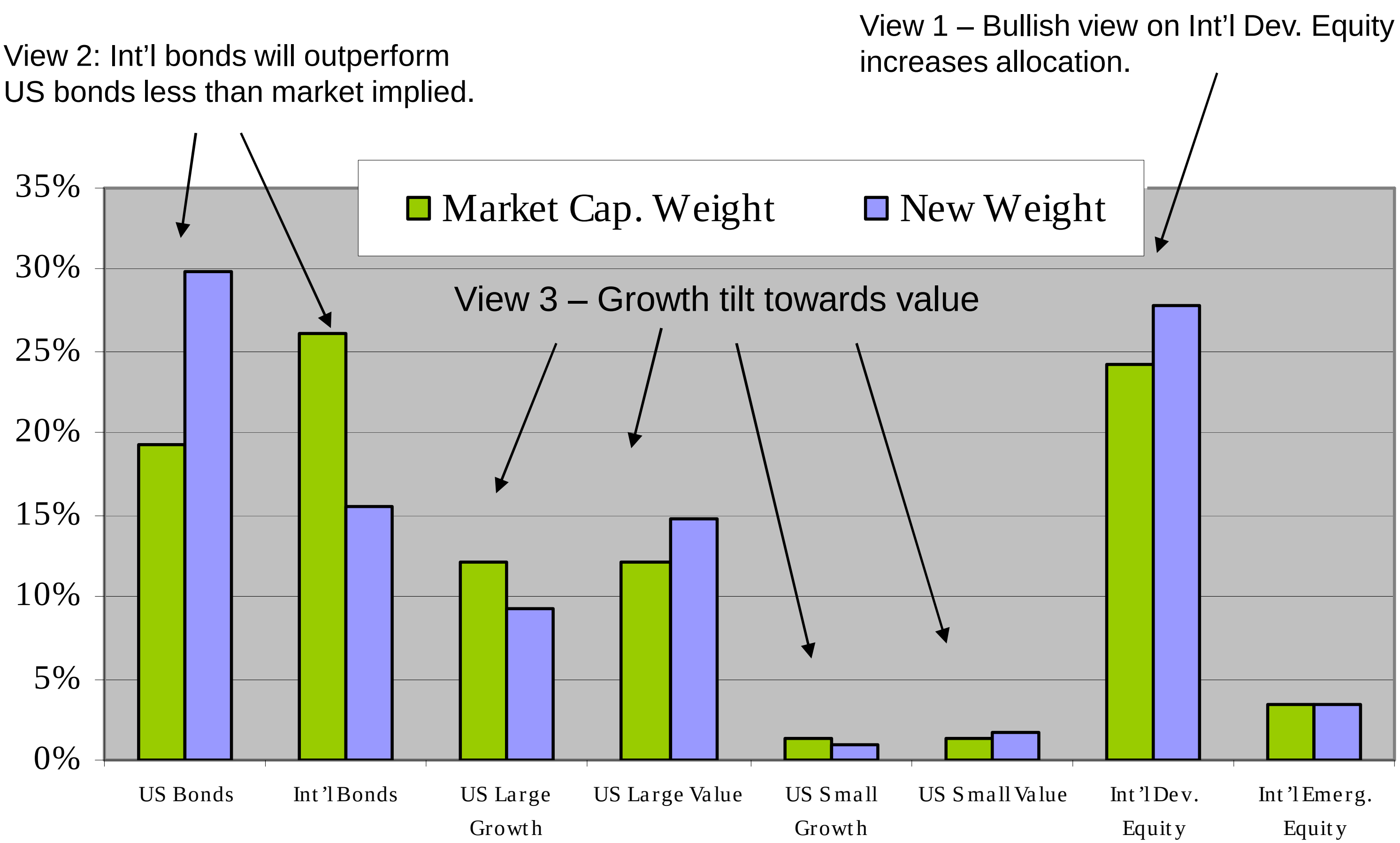
$$E[R] = \left[(\tau\Sigma)^{-1} + P'\Omega^{-1}P\right]^{-1}\left[(\tau\Sigma)^{-1}\Pi + P'\Omega^{-1}Q\right]$$

$$w = (\lambda\Sigma)^{-1}E[R]$$

COMBINED RETURN E[R] VS. EQUIL. RETURN Π



IMPACT OF VIEW: NEW ASSET ALLOCATION



RESAMPLING

- Resampling is a Monte Carlo technique for estimating the inputs for mean-variance optimization and eventually the resampled efficient frontier. It results in well diversified portfolios.

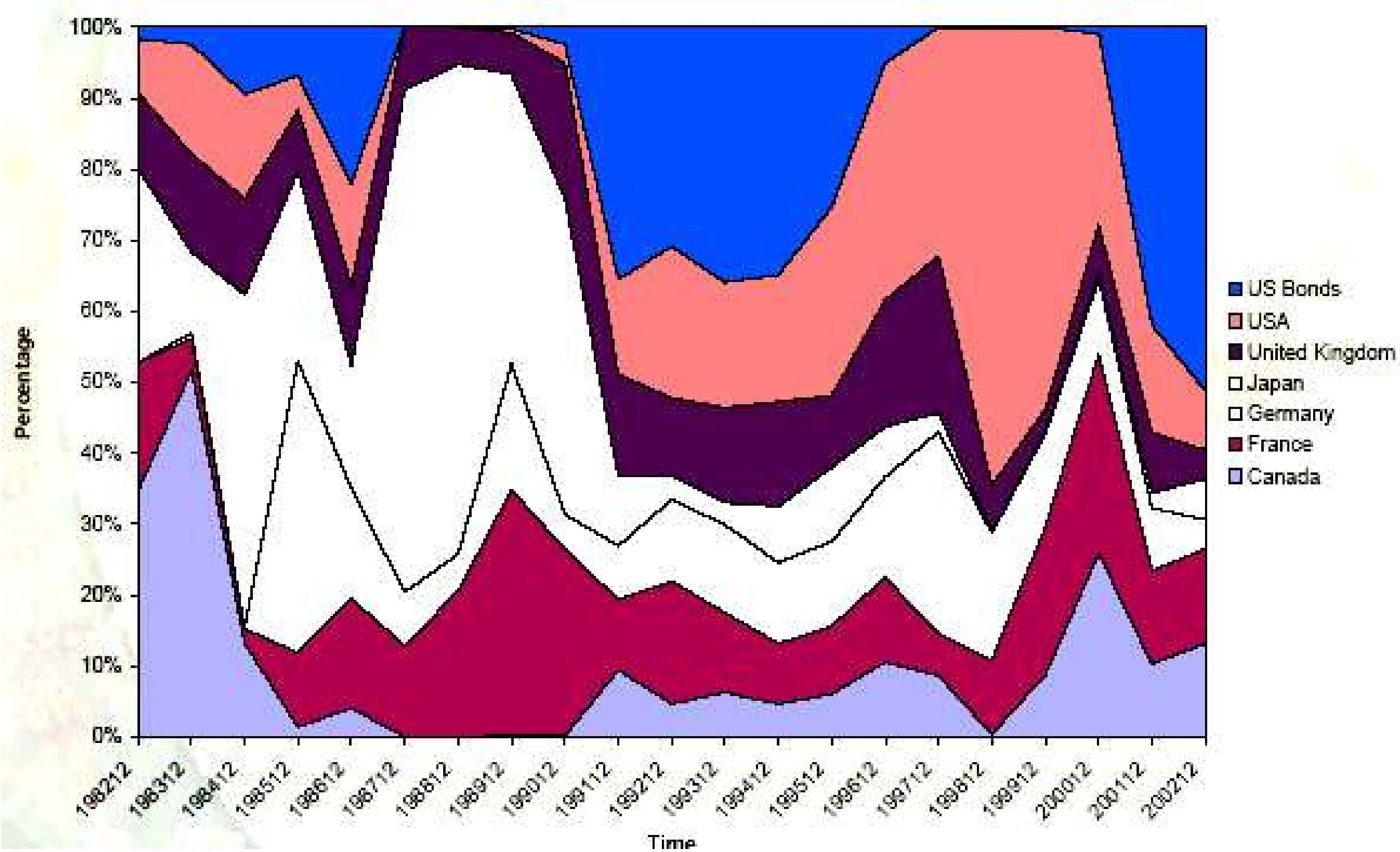
1. Estimate returns, standard deviations, and correlations
2. Run a multivariate simulation that results in a new set of returns, standard deviations, and correlations.
3. From the resulting "efficient frontier" record the weights and the returns of the efficient portfolios at predetermined standard deviation intervals (i.e. 5%, 6%, 7%, etc.)
4. Repeats Steps 2 and 3 1000+ times
5. Calculate the average allocation to the assets for each predetermined interval and the average return, and then graph them in return –standard deviation space to create the resampled frontier.

CRITICAL ISSUES

- Portfolios inherit the estimation error in the original inputs – Scherer 2002
- Lack of theory –No reason why resampled portfolios should be optimal – Scherer 2002
- In the absence of views, resampling results in active risk relative to a policy benchmark – why take bets with out a reason?
- No framework for incorporating views

RESAMPLING CRITICAL ISSUES

- Surprising large amount of variation in recommended portfolio overtime



RESAMPLING CRITICAL ISSUES

- Resampling underperformed in historical backtest

	Benchmark Returns	Implied Returns	Buy and Hold	Resampling
Total Return Statistics				
Annual Geometric Return	11.76%	11.76%	11.40%	10.67%
Annualized Standard Deviation	10.55%	10.55%	12.07%	14.98%
Realized Sharpe Ratio	0.7667	0.7667	0.6408	0.4676
Beginning Value (1/1/1983)	**$100**	**$100**	**$100**	**$100**
Ending Value (12/31/2003)	**$1133**	**$1133**	**$1066**	**$941**
Benchmark Relative Statistics				
Historical Alpha	0.00%	0.00%	-0.98%	-1.74%
Residual Risk	0.00%	0.00%	1.99%	7.83%
Active Risk	0.00%	0.00%	2.41%	8.14%
Information Ratio	0	0	-0.4926	-0.2219