ECONOMIA DEGLI INTERMEDIARI FINANZIARI AVANZATA

MODULO ASSET MANAGEMENT

LECTURE 3

MANAGEMENT APPROACH OVERVIEW

- Portfolios need maintenance and periodic revision:
  - Because the needs of the beneficiary will change
  - Because the relative merits of the portfolio components will change
  - To keep the portfolio in accordance with the investment policy statement and investment strategy

- Possible manager's choices
  - Leave the portfolio alone
  - Rebalance the portfolio
  - Asset allocation and rebalancing within the aggregate portfolio
  - Change the portfolio components
  - Indexing

- An ***active management policy*** is one in which the composition of the portfolio is dynamic
  - The portfolio manager periodically changes:
    1. The portfolio components or
    2. The components' proportion within the portfolio

- A ***passive management strategy*** is one in which the portfolio is largely left alone

MANAGEMENT APPROACH OVERVIEW

- Passive strategies:
  - Index funds:
  - holding securities, stocks and bonds, in a wide index (S&P 500) in proportion to their market value;
  - assume markets are efficient i.e. you can not beat the market
  - Markowitz Portfolio:
  - create an efficient portfolio by determining combinations of securities that maximize expected return for given risk.

- Active strategies:
  - Assume market is not efficient (random walk) i.e. can predict the market.
  - Take advantage of forecasted market movements, by shifting between cash and stocks and shifting between low and high-risk (beta) securities.
  - Analyst has the skill to select "undervalued" securities. i.e. beat the market

ACTIVE

Higher costs: trading, management, analysis

Higher confidence from the investor

Taking advantage from market inefficiencies

PASSIVE

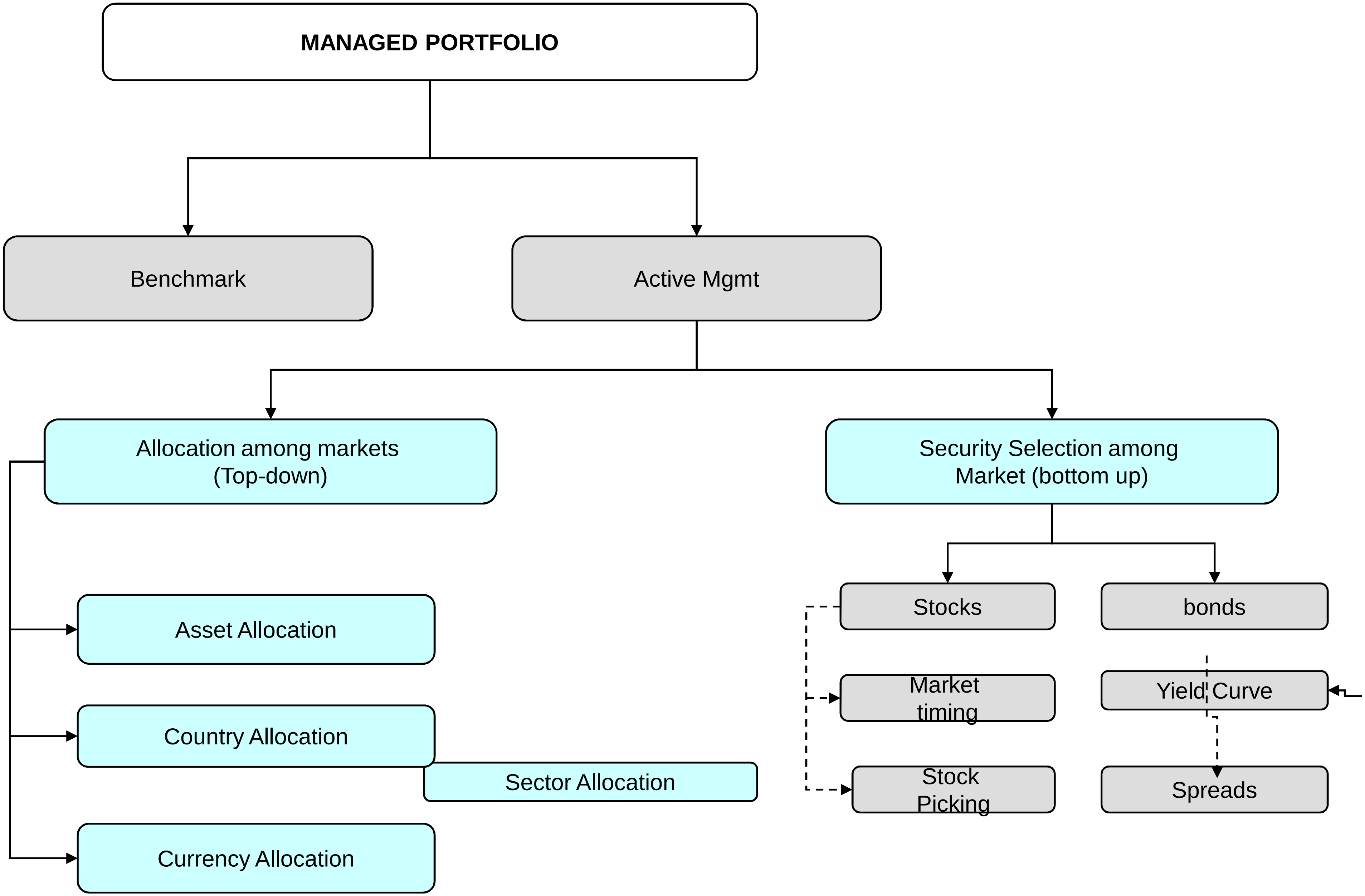
Lower costs

Higher transparency

Simple management

Difficult to understand in bear markets

TOP DOWN VS BOTTOM UP APPROACH



BUY & HOLD

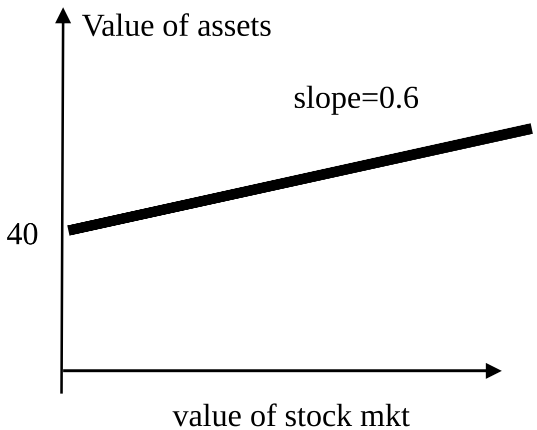
- A ***buy and hold strategy*** means that the portfolio manager hangs on to its original investments
  - The portfolio value is correlated to the stock market index
  - The ratio between Δ portfolio and Δ stock market index is exactly equal to the initial weight of stocks on the portfolio
  - The final portfolio value cannot be lower than the weight of free-risk assets
  - The potential gain is unlimited
  - The average return increases according to the stock's weight if the stock market index is higher than the risk free rate (and the opposite)

- Advantages
  Minimize operating costs
  No rebalancing and market timing activities

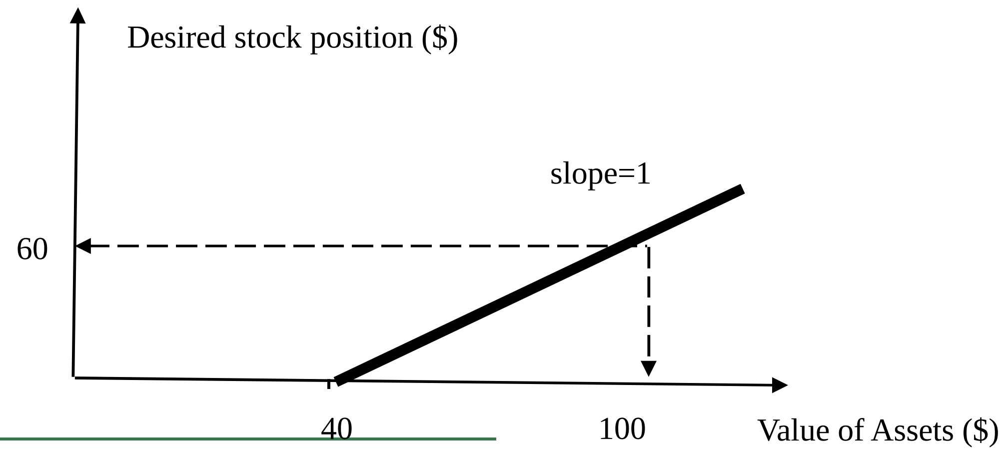
  Disadvantages
  Possible asymmetry between investors' risk tolerance and portfolio market risk exposure

- Academic research shows that portfolio managers often fail to outperform a simple buy and hold strategy on a risk-adjusted basis
  - E.g., Barber and Odean show that investors who trade the most have the lowest gross and net returns

Payoff Diagram 60/40 *stock/bill*



Exposure Diagram 60/40 *stock/bill*



PORTFOLIO REBALANCING

Rebalancing a portfolio is the process of periodically adjusting it to maintain the original conditions

Rebalancing within the Portfolio

- Constant mix strategy
- Constant proportion portfolio insurance
- Relative performance of constant mix and CPPI strategies

CONSTANT MIX

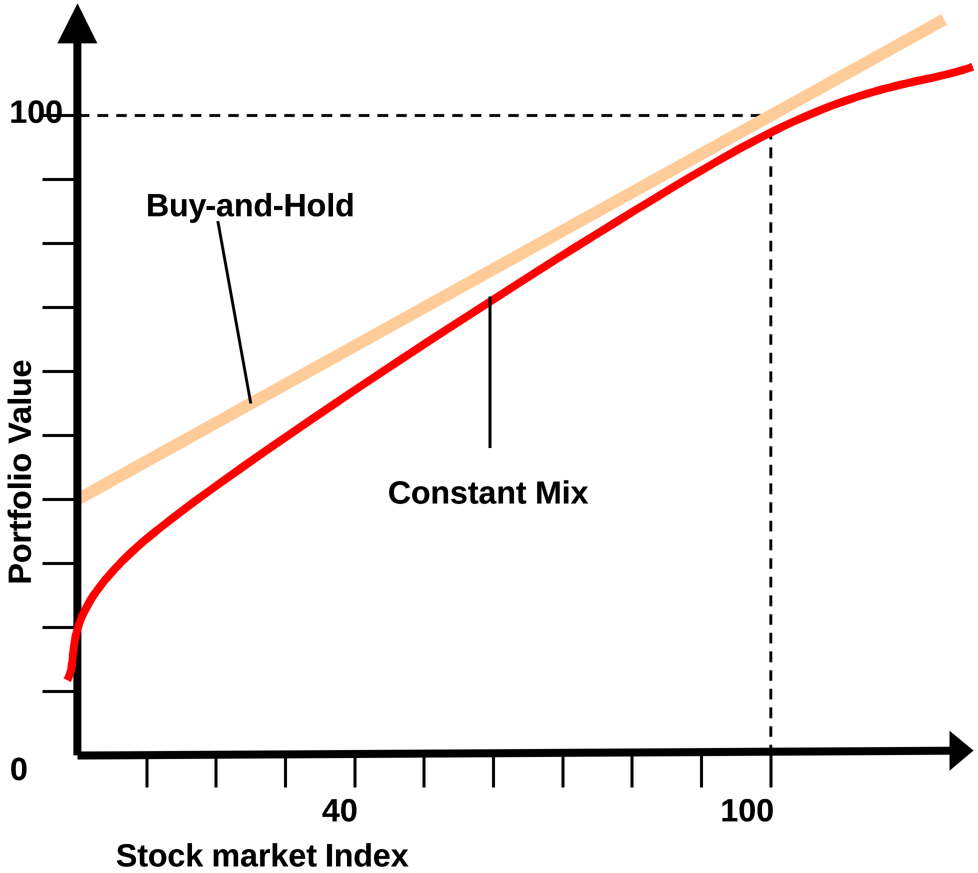
- The ***constant mix strategy***:
  - Is one to which the manager makes adjustments to maintain the relative weighting of the asset classes within the portfolio as their prices change
  - The strategic asset allocation tends to remain unchanged (useful in relative stable market)
  - Requires the purchase of securities that have performed poorly and the sale of securities that have performed the best
  - Generally, buying is required when the stock market has returns lower than the risk free rate; on the contrary, selling is required when the stock market has returns higher than the risk free rate.

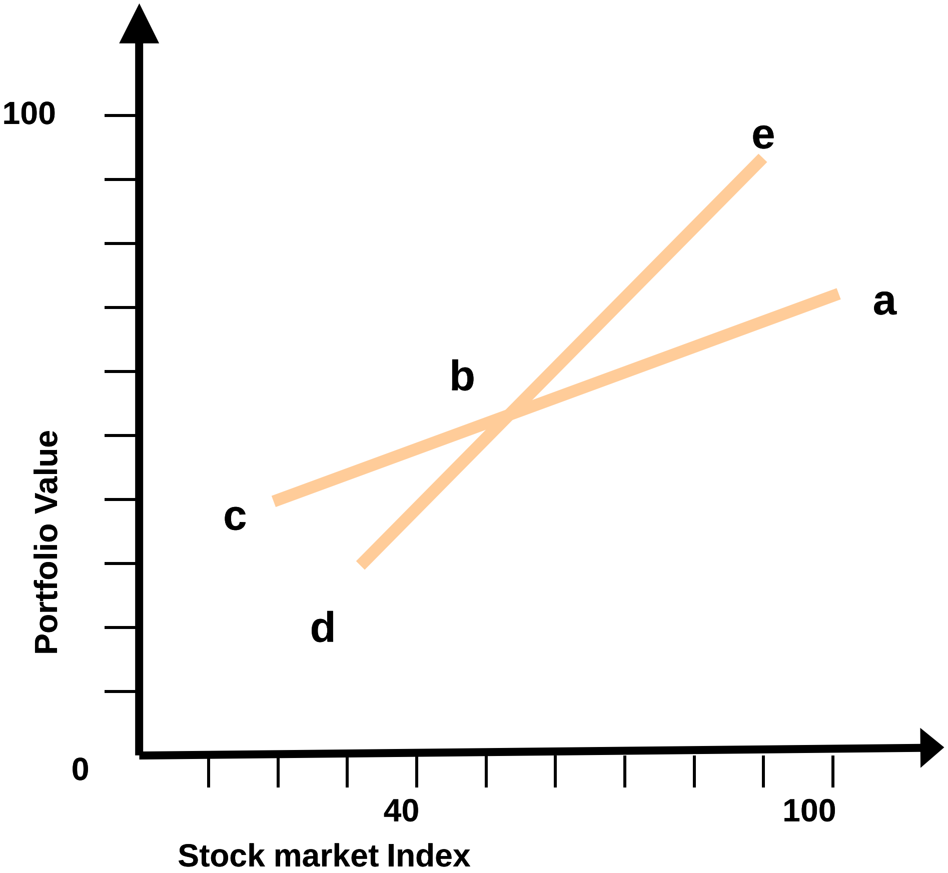
- With prolonged market trends, portfolios managed using a C-M strategy realize performances worst than those managed using a B&H strategy

  - Consider a case in which stocks fall from 100 to 90, the recover to 100. The market is flat, but it oscillates back and forth.
  - Buy-and-hold strategy - same
  - Constant-mix strategy will do better than the buy-and-hold because it buys more stocks as they falls. When shares later increases in prices, the more share purchased will enhance the return for the Constant-Mix Strategy

	Stock market	Stock value	Bond value	Portfolio Value	Stock weight
Initial	**100**	**60**	**40**	**100**	**60%**
After market∆	**90**	**54**	**40**	**94**	**57.4**
After rebalancing	**90**	**56.40**	**37.6**	**94**	**60.0**
After market∆	**110**	**66**	**40**	**106**	**62.3**
After rebalancing	**110**	**63.6**	**42.4**	**106**	**60.0**

CONSTANT MIX VS BUY & HOLD





Mkt Shock	Effects on Buy-and-Hold	Effects on Constant mix
Bear+Bull	a b a	a b e
Bear+Bear	a b c	a b d

DYNAMIC STRATEGIES

- Linear strategies (do nothing) determine linear payoff

- Dynamic strategies (do something) following the rule “buys stocks as they fall and sell stock as they rise” determine concave payoff:
  - Low elasticity in bullish markets
  - No protection floor
  - Useful in volatile markets

- Dynamic strategies (do something) following the rule “sell stocks as they fall …...” determine convex payoff:
  - High elasticity in bullish markets
  - Good protection

- Convex strategies represent the Purchase of portfolio insurance because it has a floor value;

- Concave strategies represent the sale of portfolio insurance

- Convex and concave strategies are mirror images of each other

TECNICHE DI PORTFOLIO INSURANCE

- Le tecniche gestionali rivolte all'ottenimento della protezione del capitale investito sono normalmente definite tecniche di Porfolio Insurance (PI).

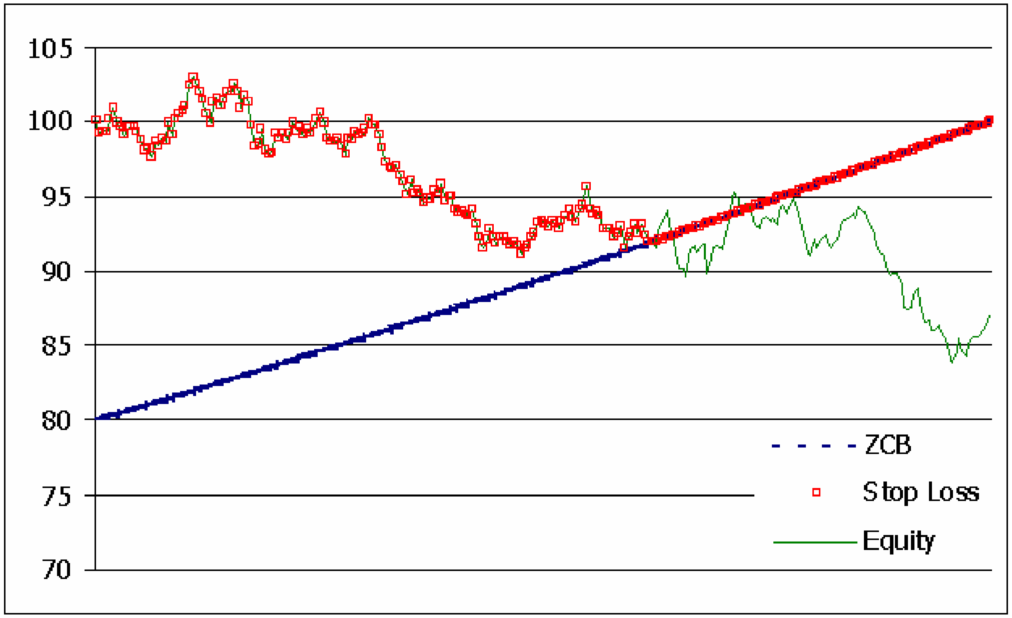
- Gli elementi principali che le caratterizzano sono:
  - Portafoglio di riferimento (indici di borsa, titoli azionari, fondi comuni ...)
  - Livello di copertura (superiore, pari o inferiore al capitale versato)
  - Orizzonte temporale della copertura (3 anni – 10/15 anni)

- Le principali tecniche di portfolio insurance sono:
  - Buy and Hold (B&H)
  - Stop Loss (SLO)
  - Protective Call Strategy (PCS)
  - Protective Put Strategy (PPS)

  - Constant Proportion (CPPI)

- La Buy & Hold strategy prevede l'investimento iniziale nel portafoglio di riferimento per un ammontare pari al residuo tra il capitale versato e il costo dello ZCB.
  - Acquisto di ZCB per un controvalore legato alle condizioni sul mercato obbligazionario del momento;
  - Acquisto di n quote di Equity per un controvalore pari al complemento a 100 del valore dello ZCB.

- La strategia di Stop Loss prevede di allocare tutto l'investimento iniziale nell'attività rischiosa. Il portafoglio rimane totalmente investito in questa attività fino a che il suo valore non scende fino a toccare il prezzo di uno ZCB avente scadenza pari a quella desiderata.



TECNICHE DI PORTFOLIO INSURANCE

- La Protective Put Strategy prevede l'investimento nella parte azionaria accompagnato all'acquisto di una opzione put. La strutturazione del portafoglio comporta la determininazione del livello di strike price della PUT compatible con la protezione del capitale investito e la successiva determinazione della quantità di opzioni da acquistare

- La Protective Call Strategy prevede l'investimento in parte nello ZCB e per il rimanente in una opzione call. Rappresenta la struttura più diffusa sul mercato per ottenere protezione dell'investito:

- Le opzioni utilizzate in queste strutture si differenziano
  - per il metodo di misurazione della performance:
    1. alla fine del periodo (tradizionale)
    2. come media di periodo (asiatica)

  - Per il sottostante oggetto di investimento:
    1. singolo asset
    2. basket di asset

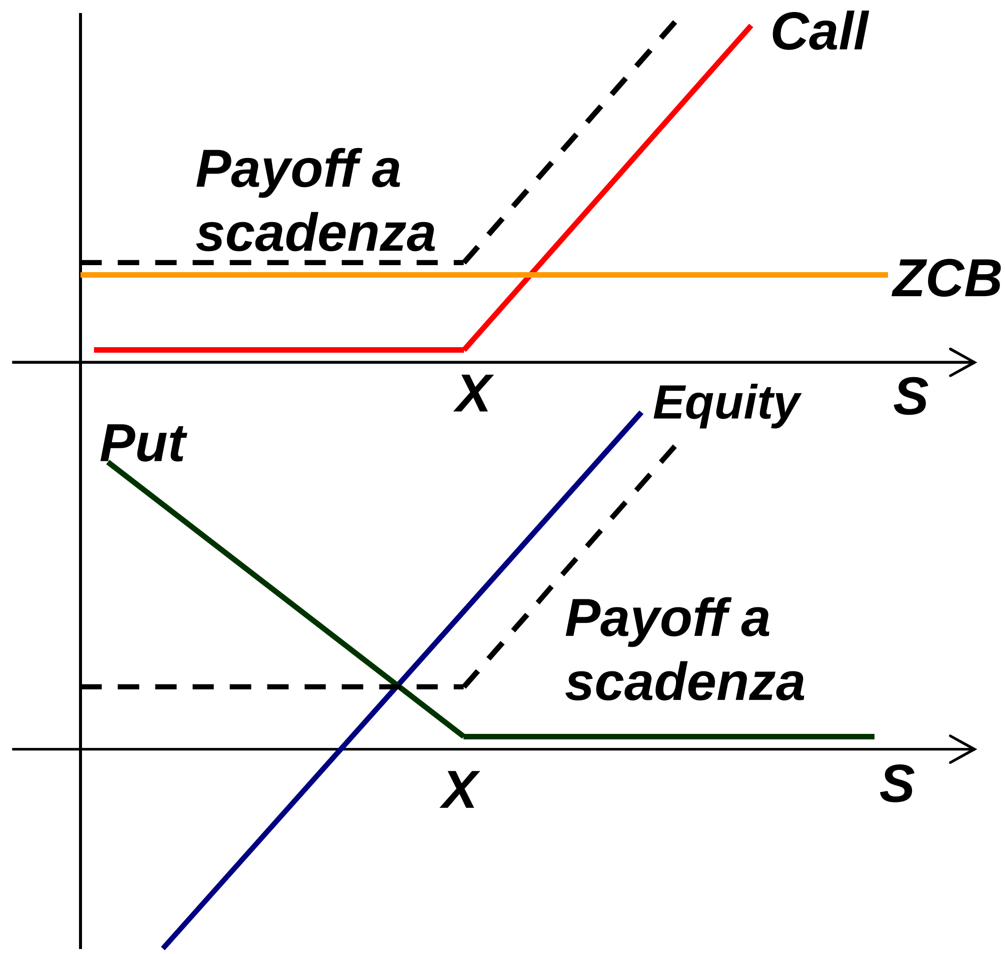
  - Per la presenza di meccanismi opzionali di path-dependency
    1. rendimento basato su classifica performance (best/worst inside basket, rainbow)
    2. barriera di attivazione/disattivazione dell'opzione (knock in/out)
    3. consolidamento dei risultati (cliquet)

- Confronto tra i pay off delle due tecniche option based

ZCB + Call

=

Put + Equity

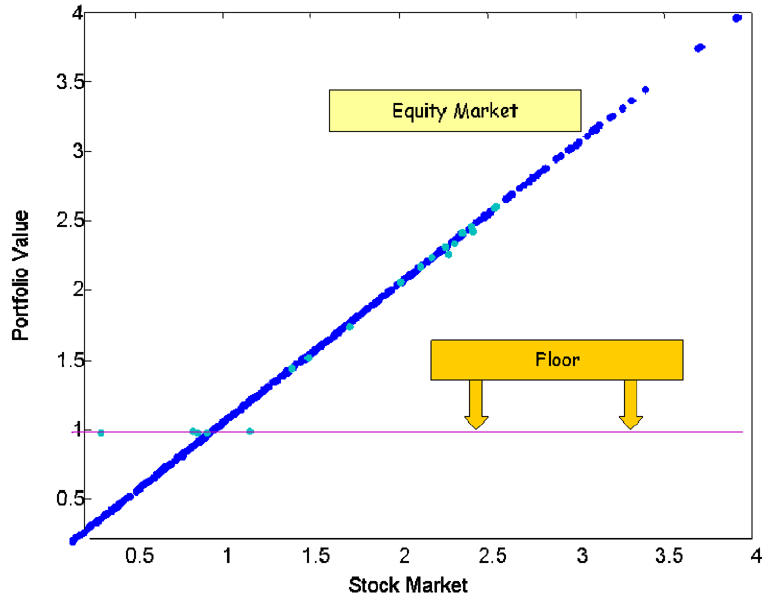


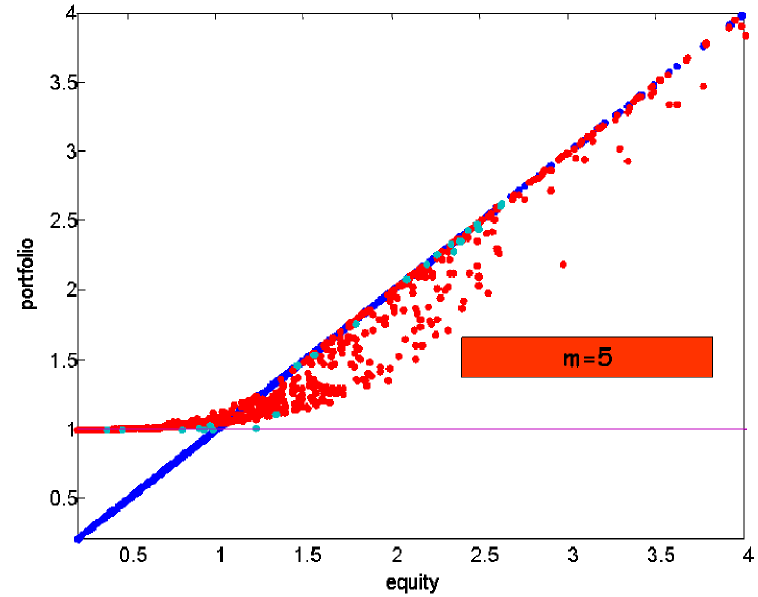
CONSTANT PROPORTION PORTFOLIO INSURANCE STRATEGIES (CPPI)

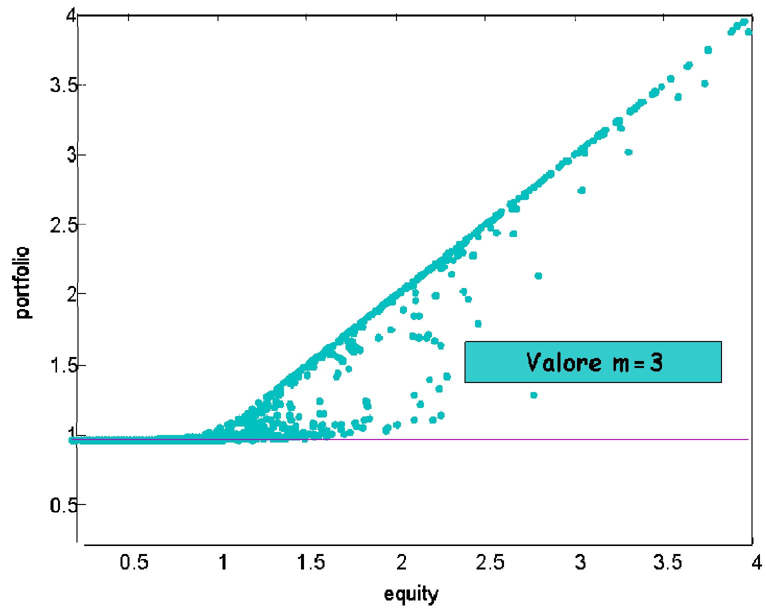
$$equity_t = m \cdot \frac{NAV_t - MIN}{NAV_t}$$

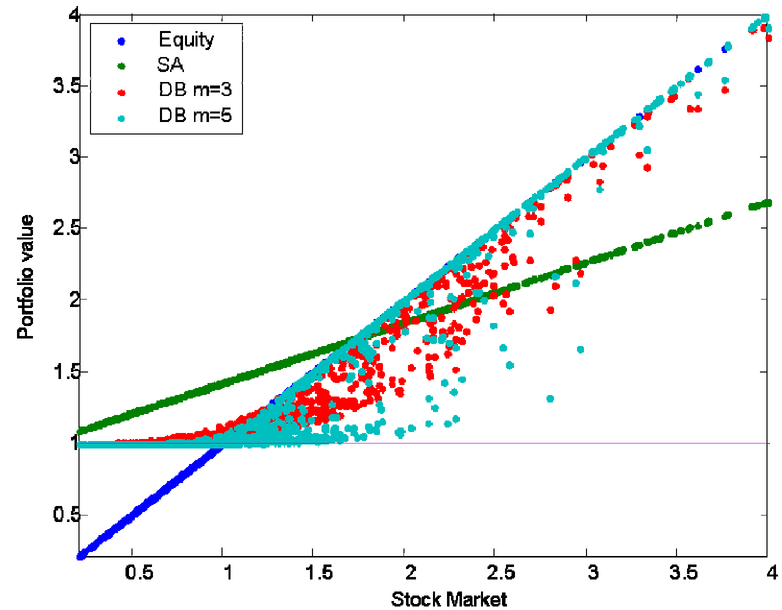
- constant proportion portfolio insurance (CPPI) strategy impone al gestore di investire una quota predeterminata del portafoglio in attività finanziarie rischiose;
- Nelle fasi di bear market, il CPPI permette una riduzione delle perdite;
- In fasi di mercato caratterizzate da elevata volatilità la strategia CPPI è tendenzialmente dannosa, in quanto vende azioni prima di un trend ascendente e compra azioni prima dell'inizio di un trend negativo.
- Il livello di m (moltiplicatore) influenza:
  - Investimento iniziale in equity
  - Livello di partecipazione alla performance del mercato azionario;
  - Costi di Ribilanciamento;
  - Probabilità di fallimento della strategia.

IMPATTO DEL MOLTIPLICATORE









DEFINIZIONE DEL RISK BUDGET: LINEE GUIDA

La definizione del Risk Budget può esser riassunta nei seguenti passaggi:

Step1: Calcolo del cuscino (C_t)

$$Ct = \frac{NAVt - LAt \cdot K}{NAVt}$$

NAV_t = Attivi del Fondo (Net Asset Value)

LA_t = Passività del fondo, ossia Present Value del Capitale e Commissioni garantite a scadenza.

K = Capitale Garantito a scadenza.

Step 2: Calcolo della leva m* compatibile con un inv. nel Risky Asset pari al 100%.

$$m_t^* = \frac{1}{Ct}$$

Step 3 : Definizione del livello di volatilità massimo (σ_{max}) corrispondente a m* :

$$\sigma_{\max} = f(m_t^*)$$

Step 4 : Definizione del buffer di rischio (b) e definizione della volatilità ottimale (σ^*)

$$\sigma_t^* = \sigma_{\max} \cdot b$$

Step 5: Definizione del target di duration (in funzione della duration delle passività del fondo) al fine di contenere le oscillazioni del cuscino dovute a movimenti nei tassi di interesse.

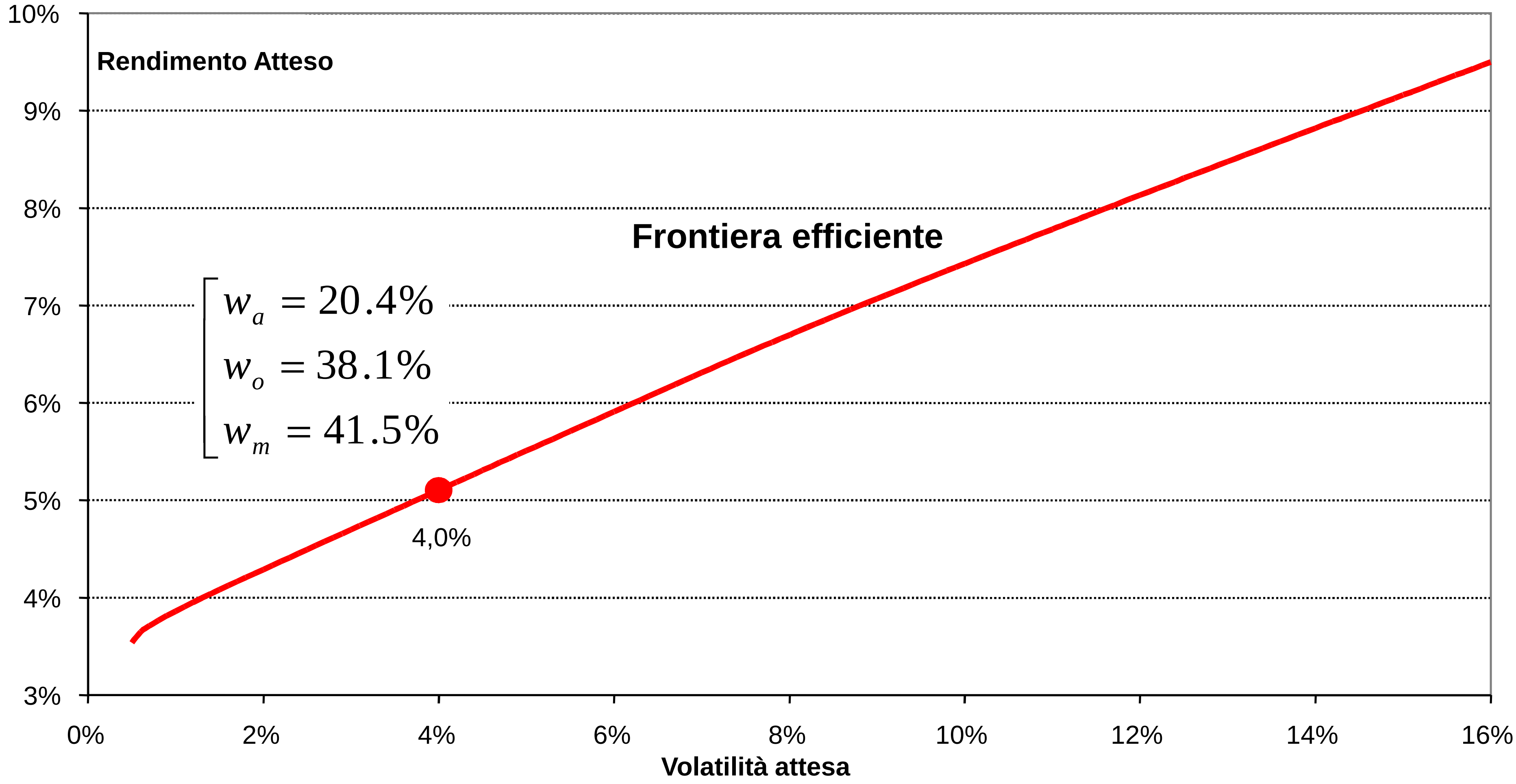
GESTIONE A VOLATILITÀ CONTROLLATA

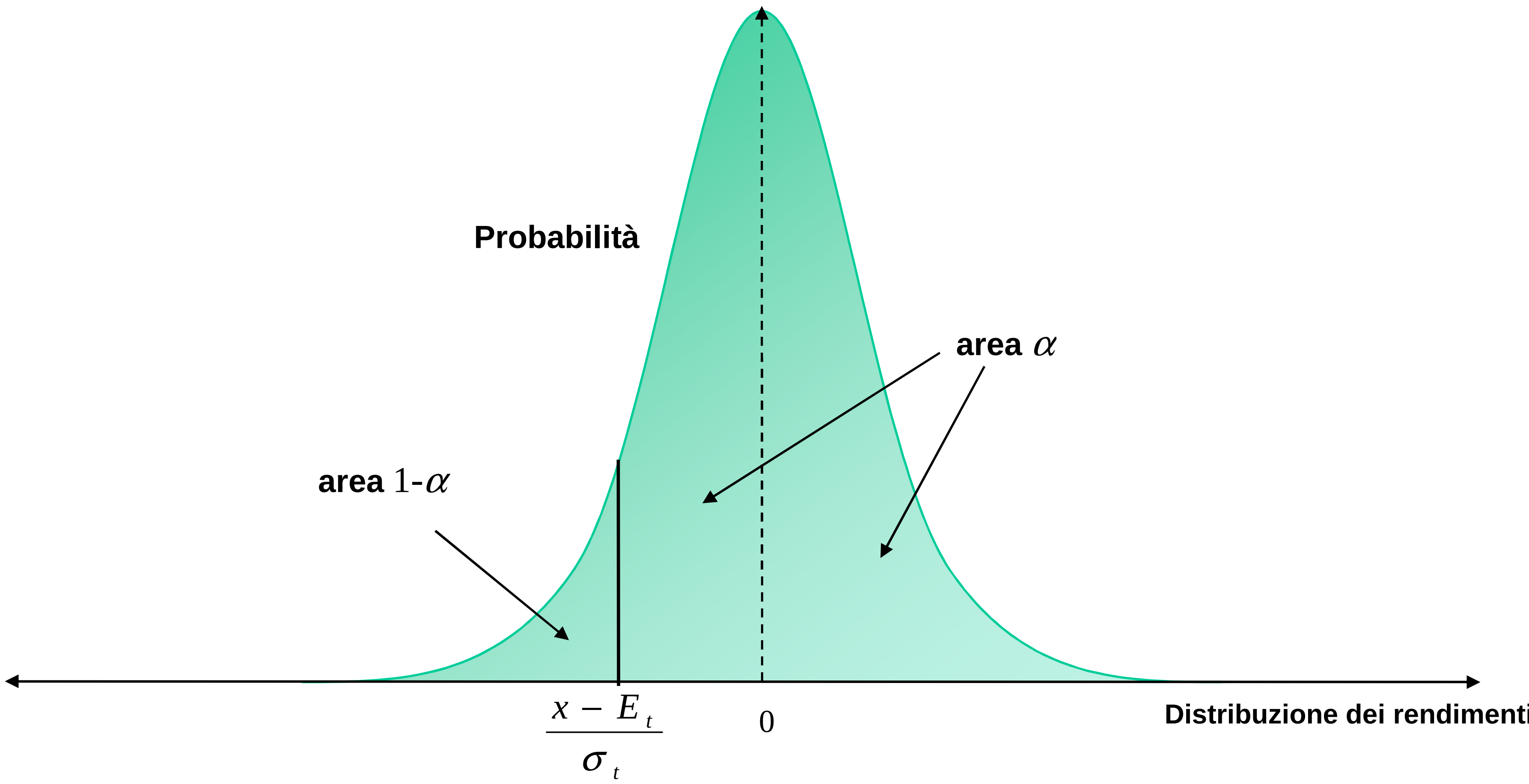
- Fondi o mandati di gestione la cui volatilità annualizzata presenta un limite massimo

- Elementi necessari per la modellizzazione dell'investimento:

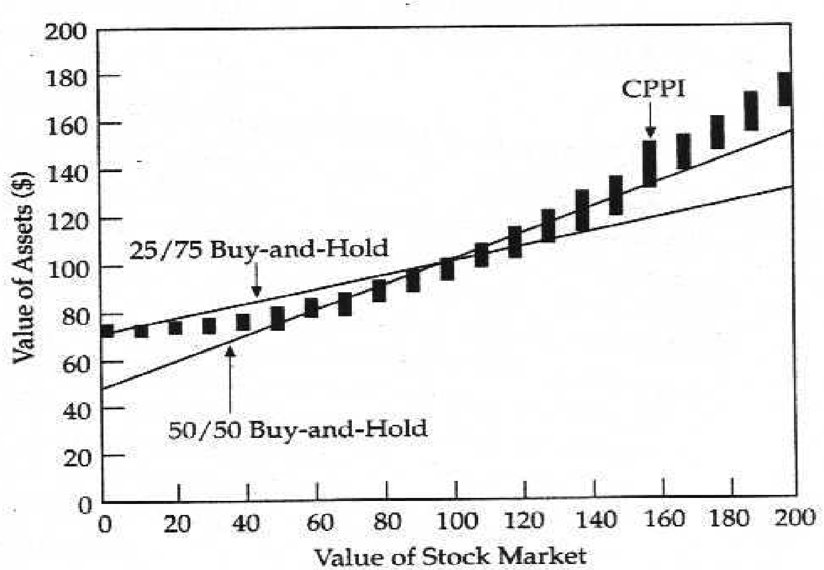
  - E_m = 3.5% rendimento atteso investimenti monetari
  - E_o = 4.5% rendimento atteso investimenti monetari obbligazionari €
  - E_a = 9,5% rendimento atteso investimenti azionari
  - σ_m = 0,5% volatilità annua attesa investimenti monetari
  - σ_o = 4% volatilità annua attesa investimenti obbligazionari €
  - σ_a = 16% volatilità annua attesa investimenti azionari
  - R = 0.3 correlazione attesa tra investimenti obbligazionari € e azionari
  - correlazione nulla tra monetario e le altre asset class

GESTIONE A VOLATILITÀ CONTROLLATA




Probabilità
area α
area $1\text{-}\alpha$
$\frac{x - E_t}{\sigma_t}$
0
Distribuzione dei rendimenti

CPPI VS BUY & HOLD



- Pay off : floor =75 and m=2

- The CPPI does not dominate buy and hold: it dispends on the market trends
- In a bull/bear market, CPPI will do well as it calls for buying/selling stocks as price rises/falls.
- Price reversals hurt CPPI investors because they sell on weakness only to see the market rebound and buy on strength only to see the market weaken.

- Three special cases:
  - If m >1, the strategy is called the constant-proportion portfolio insurance strategy (CPPI)
  - If m=1, floor= value of bills, this strategy is the buy-and-hold strategy
  - If 0<m<1, floor= 0, the strategy is the constant-mix strategy.

AT THE END ….

- In a rising market, the CPPI strategy outperforms CM
- In a declining market, the CPPI strategy outperforms CM
- In a flat market, neither strategy has an obvious advantage
- In a volatile market, the CM strategy outperforms CPPI
- The relative performance of the strategies depends on the performance of the market during the evaluation period
- In the long run, the market will probably rise, which favors CPPI; in the short run, the market will be volatile, which favors CM

REBALANCING STYLES

	Buy and Hold	Constant Mix	CPPI
Market down	**Nothing**	**Buy stock**	**Buy stock**
Market up	**Nothing**	**Sell stock**	**Buy stock**
Dynamic	**Nothing**	**High but limited by commission and costs**	**High but limited by commission and costs**
Gain potential	**Unlimited**	**Lower**	**Higher**
Portfolio minimum value	**Protected quota**	**0**	**Floor**
Risk tolerance	**0 and 1**	**Higher**	**Lower**
Better situation		**Volatility**	**Trend**
	Static Strategy	**Concave payoff**	**Convex Payoff**

CORE & SWING PORTFOLIO

- Semi-active Management Style obtained by dividing the managed portfolio into 2 blocks:
  - Core Portfolio (Strategic)
  - Swing portfolio (Tactical) - mispriced securities

- Dimension of core portfolio depends on:
  - EMH on benchmark
  - Max underperformance tolerance
  - Expected forecasting ability
  - Tracking Error aversion

- Benefit
  - MAX – Bench = E(R) >0

Cost

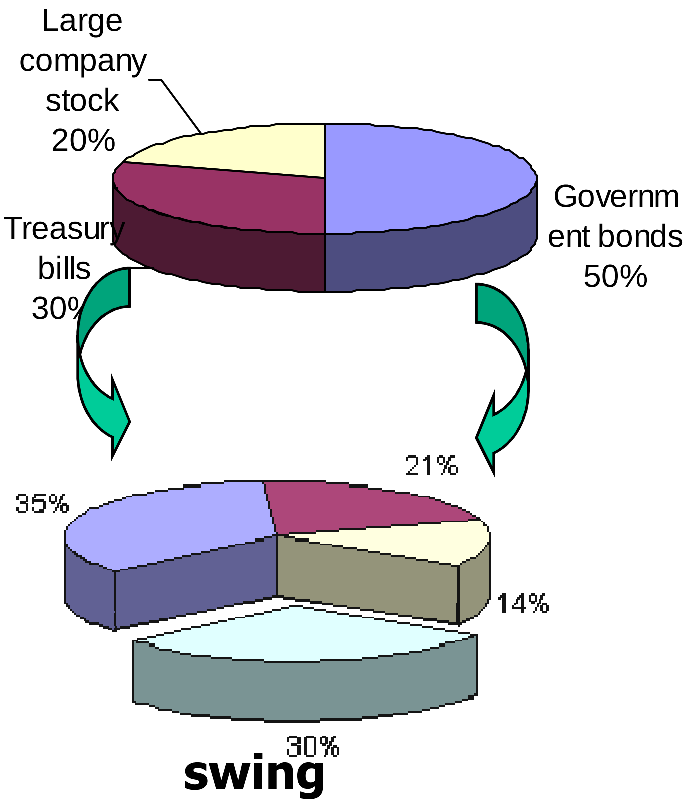
MIN – Bench = E(R) < 0

- Rswing = MAX (f) + MIN (1-f)

- Break even frequency level

$$F = \frac{Bench - MIN}{MAX - MIN}$$

EXAMPLE: CORE 70% SWING 30%



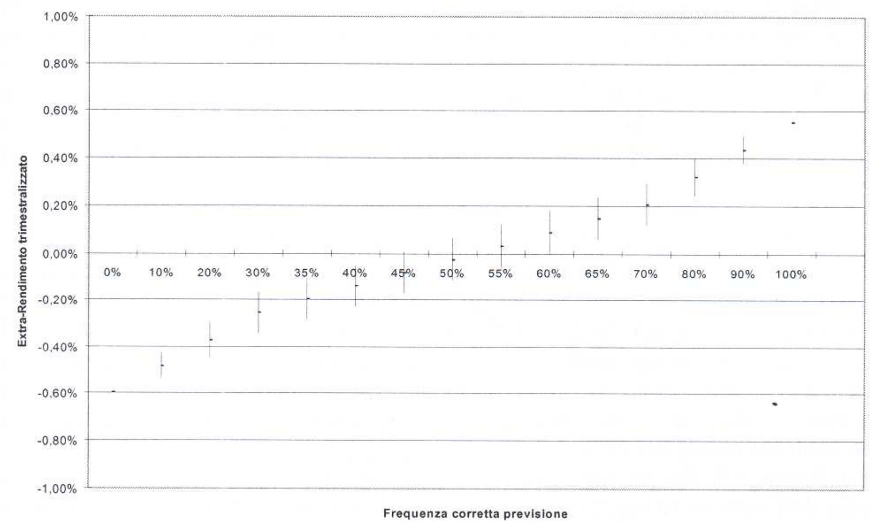
	Benchmark	*Sampling al 70%*	Core
Government bond	50 (%)		35 (%)
Treasury bills	30		21
Large company stocks	20		14

CORE & SWING SIMULATION RESULT

Tabella 1.6 Market timing: extra-rendimenti mensili trimestralizzati (benchmark: 60% D.J. Eurostoxx 50 e 40% Bond 2 anni)

Swing	Frequenza di corretta previsione (%)														
(%)	0	10	20	30	35	40	45	50	55	60	65	70	80	90	100
5	-0.30	-0.24	-0.19	-0.13	-0.10	-0.07	-0.04	-0.01	0.02	0.04	0.07	0.10	0.16	0.22	0.27
10	-0.60	-0.48	-0.37	-0.26	-0.20	-0.14	-0.08	-0.03	0.03	0.09	0.15	0.21	0.32	0.43	0.55
15	-0.90	-0.72	-0.56	-0.38	-0.30	-0.21	-0.12	-0.04	0.05	0.13	0.22	0.31	0.48	0.65	0.83
20	-1.19	-0.97	-0.74	-0.51	-0.39	-0.28	-0.16	-0.05	0.06	0.18	0.30	0.41	0.64	0.87	1.10
25	-1.49	-1.21	-0.93	-0.64	-0.49	-0.35	-0.20	-0.06	0.08	0.22	0.37	0.51	0.80	1.09	1.38
30	-1.79	-1.45	-1.11	-0.77	-0.59	-0.42	-0.25	-0.08	0.10	0.27	0.44	0.62	0.97	1.31	1.66
35	-2.08	-1.69	-1.30	-0.89	-0.69	-0.49	-0.29	-0.09	0.11	0.31	0.52	0.72	1.13	1.53	1.93
40	-2.38	-1.92	-1.48	-1.02	-0.79	-0.56	-0.33	-0.10	0.13	0.36	0.59	0.82	1.29	1.74	2.21
45	-2.67	-2.16	-1.66	-1.15	-0.89	-0.63	-0.37	-0.11	0.15	0.40	0.67	0.93	1.45	1.96	2.49
50	-2.96	-2.40	-1.85	-1.27	-0.98	-0.70	-0.41	-0.13	0.16	0.45	0.74	1.03	1.61	2.18	2.77
60	-3.55	-2.88	-2.21	-1.53	-1.18	-0.84	-0.49	-0.15	0.19	0.54	0.89	1.24	1.94	2.62	3.33
70	-4.13	-3.35	-2.58	-1.78	-1.38	-0.98	-0.57	-0.18	0.23	0.63	1.04	1.44	2.26	3.07	3.89
80	-4.71	-3.82	-2.94	-2.03	-1.57	-1.12	-0.65	-0.20	0.26	0.72	1.18	1.65	2.59	3.51	4.46
90	-5.29	-4.29	-3.31	-2.29	-1.77	-1.26	-0.73	-0.23	0.29	0.81	1.33	1.86	2.91	3.95	5.03
100	-5.87	-4.76	-3.67	-2.54	-1.96	-1.40	-0.82	-0.25	0.32	0.90	1.48	2.07	3.24	4.40	5.59

Figura 1.7 Market timing: extra-rendimenti mensili trimestralizzati - Swing 10% (benchmark: 60% D.J. Eurostoxx 50 e 40% Bond 2 anni)



HOW DO YOU MEASURE THE RISK OF A PORTFOLIO?

- Absolute risk: Volatility of returns

- Relative risk: Volatility relative to a benchmark
  - Example: Tracking error or Active risk

- Active return = Portfolio return - Benchmark return

- Tracking error = standard deviation of active returns

  - Application: Information ratio = alpha / tracking error
    i. (Alpha = average active return)

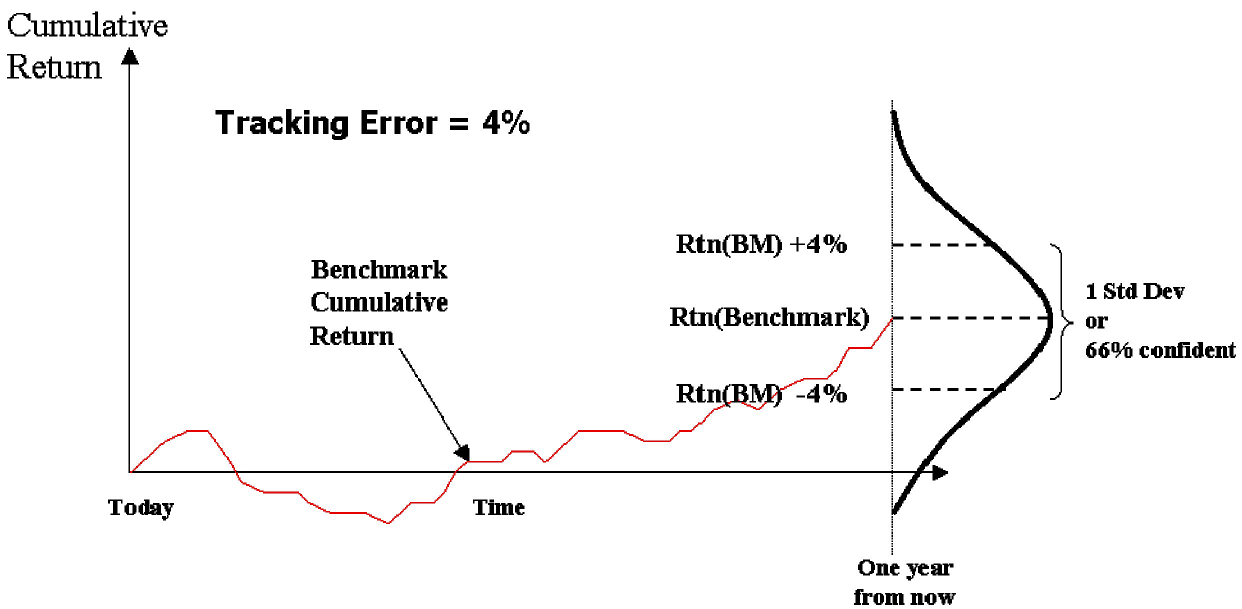
- Estimation of tracking error:
  - Trailing active returns (backward looking estimate)
  - Risk model e.g., Barra (forward looking estimate)

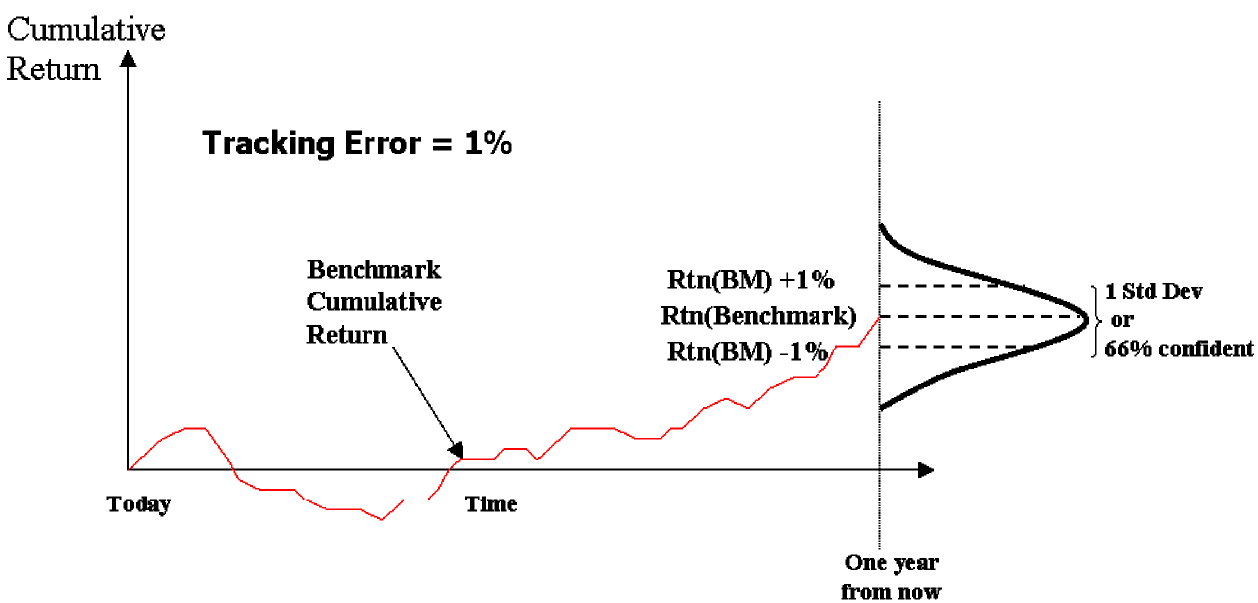
A MEASURE OF RISK

- An important risk metric for portfolios that are managed versus a benchmark
- Typical values:
  - 0% for index funds
  - less than 2% for enhanced index funds
  - 5% for active large cap stock funds

- Number of stocks held - those in the benchmark and those not in the benchmark
  - Size or Style or Sector bets
  - Beta
  - Benchmark volatility

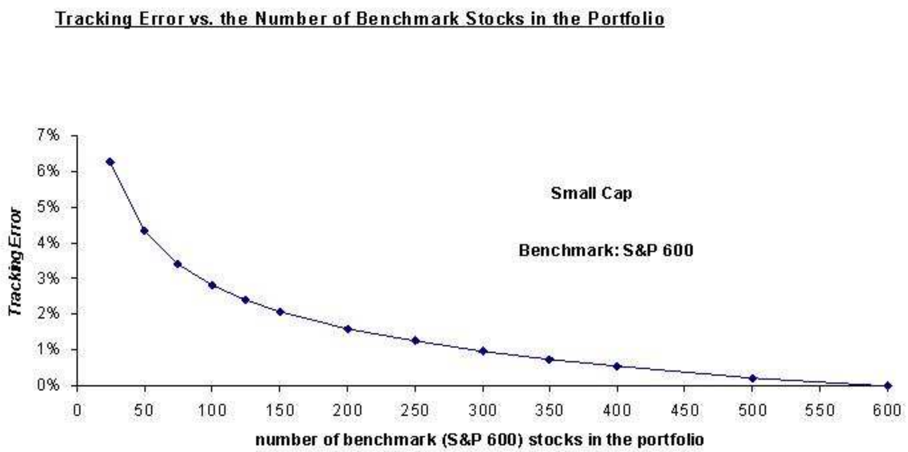
TEV & DISTRIBUTION OF RETURN

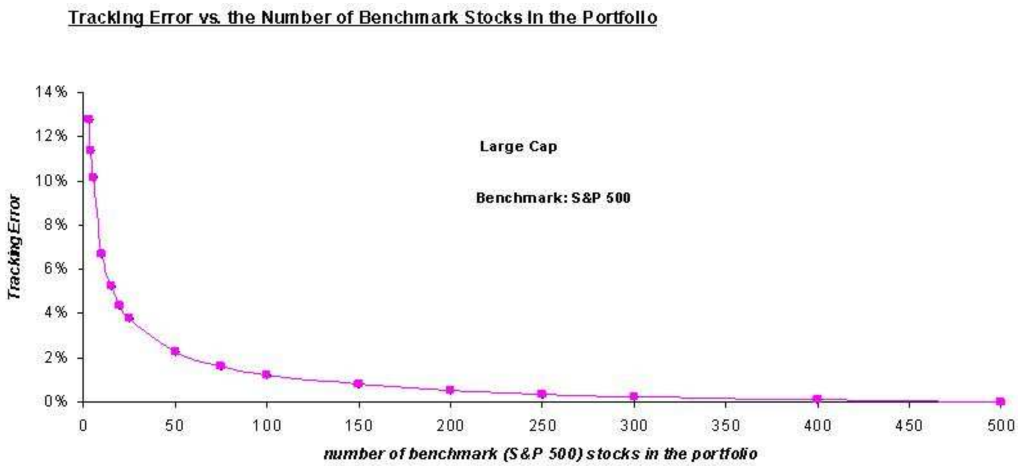


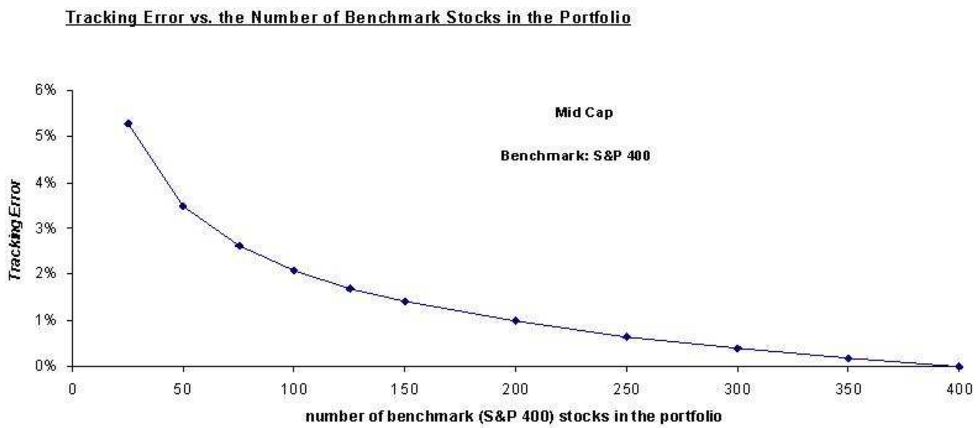


EFFECT OF NUMBER OF STOCKS

- Tracking error falls as the portfolio includes more and more of the stocks in the benchmark
- An optimally constructed portfolio of just 50 stocks can track the S&P 500 within 2%
- Tracking error rises as the portfolio starts to include stocks that are not in the benchmark

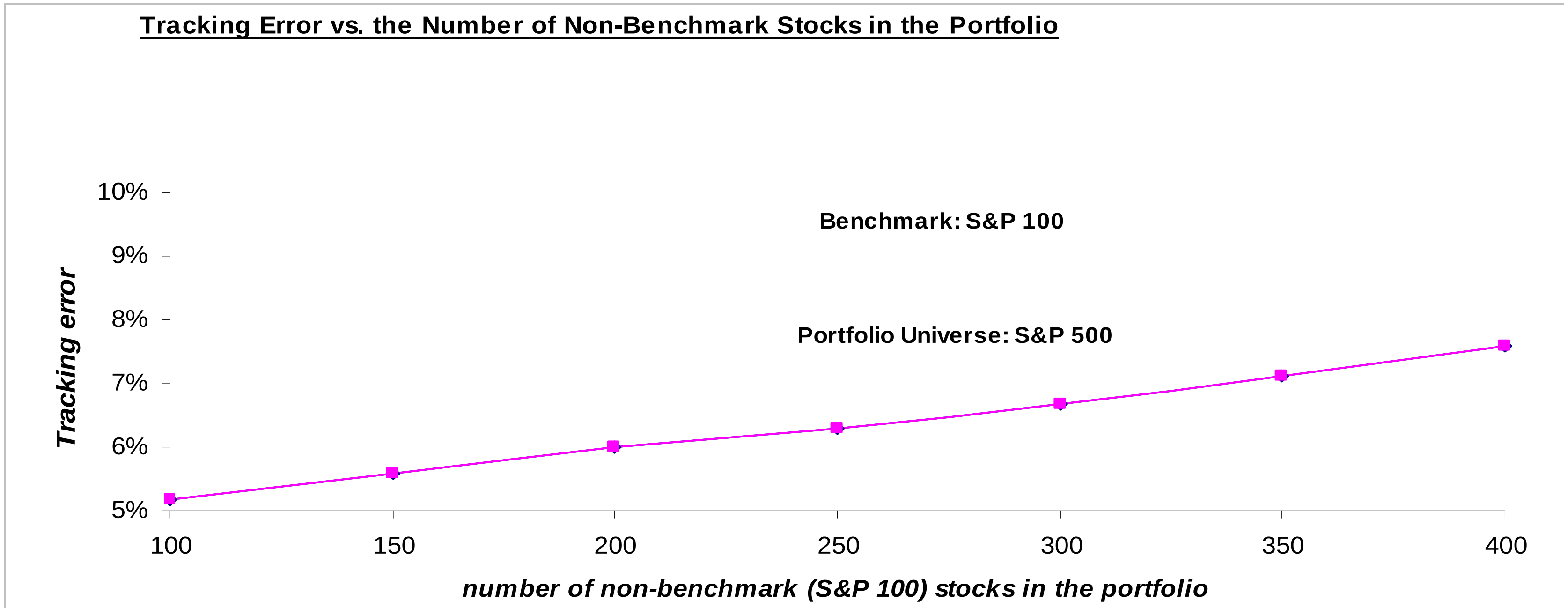






NON BENCHMARK STOCK: IMPACT ON TEV

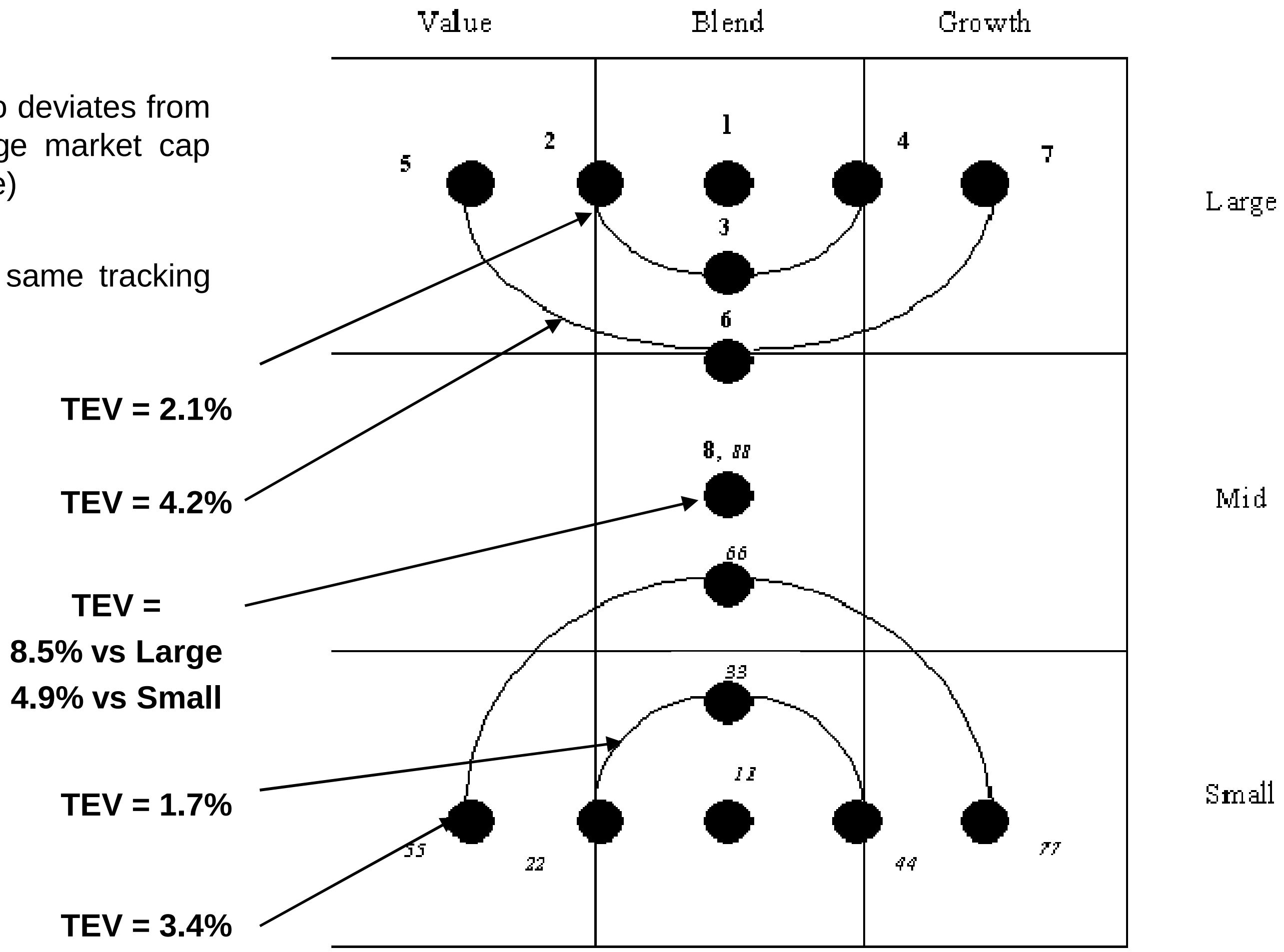
- Tracking Error Rises With the Increase in Non-Benchmark Stocks



Note: All of the S&P 100 stocks are present in the S&P 500. We start with a portfolio that has all 100 of the stocks in the S&P 100 index and progressively add to it stocks that are not in the S&P 100 index but are in the S&P 500 index. The tracking error for such a portfolio versus the S&P 100 index is shown above. So, for example, when the portfolio has 200 of the S&P 500 stocks in addition to the S&P 100 stocks (i.e., 300 in all) then its tracking error, upon optimal choice, is 6% as shown above.

EFFECT OF SIZE AND STYLE

- Tracking error rises as the portfolio deviates from its benchmark in terms of average market cap (size) or investment valuation (style)
- Different portfolios can have the same tracking error



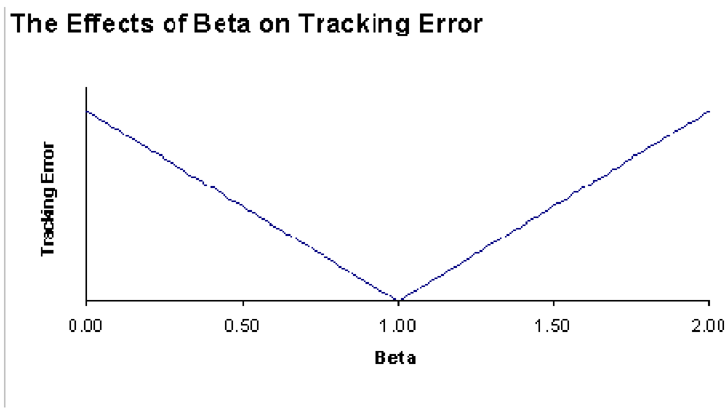
Investment valuation is along the horizontal axis and market cap (size) is along the vertical axis.

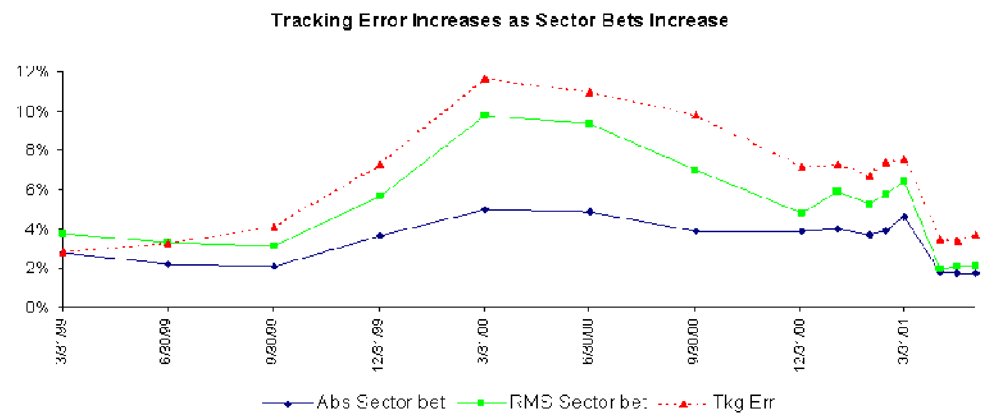
Large Cap
Portfolio 1 has a tracking error of 0%. Portfolios 2, 3, 4 have nearly similar tracking errors of around 2.1%. Portfolios 5, 6, 7 have nearly similar tracking errors of around 4.2%. Portfolio 8 has a tracking error of 8.5%. All of the above tracking errors are versus the S&P 500, the large cap index.

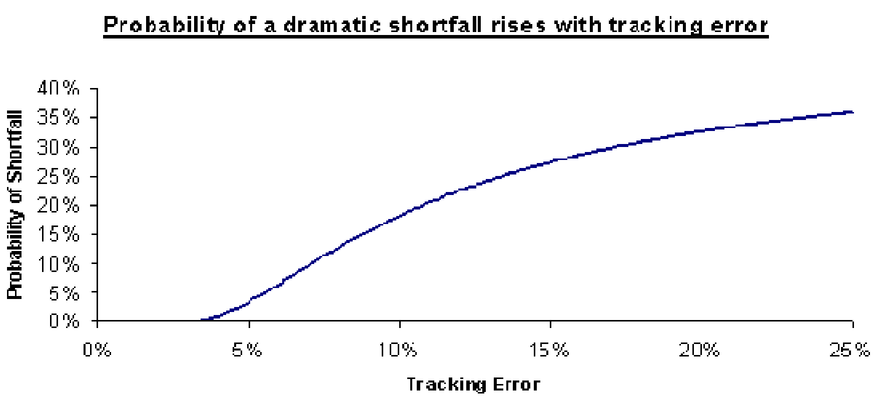
Small Cap
Portfolio *11* has a tracking error of 0%. Portfolios *22, 33, 44* have nearly similar tracking errors of around 1.7%. Portfolios *55, 66, 77* have nearly similar tracking errors of around 3.4%. Portfolio *88* has a tracking error of 4.9%. All of these tracking errors are versus the S&P 600, the small cap index.

EFFECT OF SECTOR BETS AND BETA

- Tracking error rises as the portfolio's sector allocations begin to differ from those of the benchmark
- Tracking error rises as the portfolio's beta with respect to the benchmark begins to differ from 1
- Holding cash decreases a portfolio's overall volatility but increases its tracking error
- Probability of dramatic shortfall (active return < -10%) and outperformance rises with tracking error







To better understand stock alphas assume that for a benchmark B (could be the market) the model without intercept holds:

$$r_P = \beta_P r_B + \varepsilon_P ,$$

where, estimated from a model with intercept

$$\beta_P = \frac{\mathrm{cov}(r_P, r_B)}{\sigma_B^2}.$$

Then the investor can usually observe that

$$\mu_P = E[r_P] = \beta_P E[r_P] + E[\varepsilon_P] = \beta_P \mu_B + \alpha_P ,$$

where $\boxed{\alpha_P \neq 0}$ while CAPM says $\boxed{\alpha_P = 0}$

Of course the investor hopes and tries to achieve $\boxed{\alpha_P > 0}$

Active returns and active weights are given by

$$r_{PA} = r_P - r_B,$$
$$\mathbf{w}_{PA}^T \mathbf{r} = \mathbf{w}_P^T \mathbf{r} - \mathbf{w}_B^T \mathbf{r}.$$

The corresponding active variance has two representations:

$$\sigma_{PA}^2 = \sigma_P^2 + \sigma_B^2 - 2\,\mathrm{cov}(\mathbf{r}_P, \mathbf{r}_B),$$
$$\sigma_{PA}^2 = \mathbf{w}_{PA}^T \mathbf{\Omega} \mathbf{w}_{PA}.$$

An active beta is defined in an analog way

$$\beta_{PA} = \frac{\mathrm{cov}(\mathbf{r}_{PA}, \mathbf{r}_B)}{\sigma_B^2} = \frac{\mathrm{cov}(\mathbf{r}_P - \mathbf{r}_B, \mathbf{r}_B)}{\sigma_B^2},$$
$$= \frac{\mathrm{cov}(\mathbf{r}_P, \mathbf{r}_B)}{\sigma_B^2} - \frac{\mathrm{cov}(\mathbf{r}_B, \mathbf{r}_B)}{\sigma_B^2} = \beta_B - 1.$$

ACTIVE RETURN MODEL

The active returns model written in terms of active beta is

$$r_{PA} = \beta_{PA} r_B + \tilde{\varepsilon}_{PA},$$

with active variance (TEV)

$$\sigma^2_{PA} = \mathrm{var}(r_{PA}) = \beta^2_{PA}\sigma^2_B + \sigma^2_\varepsilon, \text{ since } corr(r_B, \varepsilon_{PA}) = 0.$$

Note:

(1) Active variance is often called Tracking Error Variance (TEV)

(2) Performance of portfolio manager is measure by TEV (low=good)

(3) TEV is minimal, if active beta $\beta_{PA} = 0.$

(4) Since $\beta_{PA} = \beta_P - 1$ a portfolio beta of $\beta_P = 1$ implies $\beta_{PA} = 0.$

(5) Usually managers avoid $\beta_P > 1.$

INFORMATION RATIO

The most important performance measure of a portfolio manager is the *Information Ratio",*

Goal of a portfolio manager is to maximize

$$IR = \frac{\alpha_P}{\sigma_B}$$

Grinold, Kahn: Fundamental law of Active Management

$$E(\alpha) = \sigma_\alpha * IC * \sqrt{N}$$

IC: corr(Expected alpha, realized alpha)

A new manager achieves

$$\alpha_P = 3.5\%, \ \sigma_B = 5.5\% \Rightarrow IR = \frac{\alpha_P}{\sigma_B} = 0.64\%.$$

How good is her IR?

Compare to others:

%	90	75	50	25	10	
IR		1.0	0.5	0	-0.5	-1.0

hire → fire

INFORMATION RATIO

Referring to single stock (security, cluster of securities, asset class) Grinold proposed

$$\alpha_i = \sigma_{\alpha i} * IC * S_i$$

α_{ι}: : historical volatility of alpha generated on stock I
S_i : confidence level of forecasting

Assuming that corr(α_{ι} ; α_j)=0 we can determine the optimal stock bet as follow:

$$W_i - w_i = \frac{\alpha_i}{\sigma_{\alpha i}^2} \frac{1}{2\lambda} = \frac{IC\sigma_{\alpha i} S_i}{\sigma_{\alpha i}^2} \frac{1}{2\lambda} = \frac{ICS_i}{\sigma_{\alpha i} \, 2\lambda}$$

ALFA ESTIMATION

Alphas are estimated together with the portfolio betas, using the following model:

$$r_{P,t} = \alpha_P + \beta_P r_{B,t} + \varepsilon_{P,t}, \;\; t = 1, \ldots, T.$$

Since this can also be written as

$$\begin{aligned} r_{P,t} &= \alpha_P + \beta_P r_{B,t} + (\tilde{\tilde{\varepsilon}}_{P,t} - \alpha_P) \\ &= \beta_P r_{B,t} + \tilde{\varepsilon}_{P,t}, \end{aligned}$$

one can state that an existing significant intercept/alpha in the first equation, or a non-zero error term in the second equation are returns not explained by the CAPM model

VALUE ADDED

To estimate the value added by a manager, Grinold and Kahn propose

$$r_P = \beta_P r_B + \varepsilon_P$$

with expected returns

$$\mu_P = \beta_P \mu_B + \alpha_P = (\beta_{PA} + 1)\mu_B + \alpha_P .$$

Define

$$\mu_B = \overline{\mu_B} + \Delta\mu_B ,$$

with $\overline{\mu_B}$ the long-term average (consensus return) and $\Delta\mu_B$ the local variation of benchmark mean (benchmark timing), then

$$\mu_P = \mu_B + \beta_{PA}\overline{\mu}_B + \beta_{PA}\Delta\mu_B + \alpha_P$$

Actual benchmark expected excess return

Return due to active management and consensus return

Return due to active management and exceptional forecasting (benchmark timing)

Return due to stock alphas and stock selection

VARIANCE DECOMPOSITION

The equivalent expression holds for the variance

$$\sigma_P^2 = (1+\beta_{PA})^2 \sigma_B^2 + \sigma_\varepsilon^2$$
$$= \sigma_B^2 + 2\beta_{PA}\sigma_B^2 + \beta_{PA}^2\sigma_B^2 + \sigma_\varepsilon^2$$

Actual benchmark variance

Covariance due to active betas

Covariance due to active betas

Covariance due to stock selection

Taking the difference:

$$\mu_P - \lambda_T\sigma_P^2 = \mu_B - \lambda_T\sigma_B^2 +$$

$$\beta_{PA}(\bar{\mu}_B - 2\lambda_T\sigma_B^2) +$$

$$\beta_{PA}(\Delta\mu_B - \lambda_T\beta_{PA}\sigma_B^2) +$$

$$\alpha_P - \lambda_T\sigma_\varepsilon^2$$

"All forecast, no action"

"Action, no forecast"

} "cannot change"

"Forecast, action"

"Forecast, action"

} "value added"

➔ Value added:

$$VA = \beta_{PA}(\Delta\mu_B - \lambda_T\beta_{PA}\sigma_B^2) + \alpha_P - \lambda_T\sigma_\varepsilon^2$$

NOTION OF TRACKING ERROR

When managing an active investment portfolio against a well-defined benchmark (such as the Standard & Poor's 500 or the IPSA index), the goal of the manager should be to generate a return that exceeds that of the benchmark while minimizing the portfolio's return volatility relative to the benchmark. Said differently, the manager should try to maximize **alpha** while minimizing **tracking error**.

Tracking error can be defined as the extent to which return fluctuations in the managed portfolio are *not correlated* with return fluctuations in the benchmark. The concept is analogous to the statistic $(1 - R^2)$ in a regression context.

A flexible and straightforward way of measuring tracking error can be developed as follows:

Let: w_i = investment weight of asset i in the managed portfolio
R_{it} = return to asset i in period t
R_{bt} = return to the benchmark portfolio in period t.

With these definitions, we can define the period t return to managed portfolio as:

$$R_{pt} = \sum_{i=1}^{N} w_i R_{it}$$

where:

N = number of assets in the managed portfolio

and:

$$\sum_{i=1}^{N} w_i = 1$$ (i.e., the managed portfolio is fully invested).

We can then specify the period t *return differential* between the managed portfolio and the benchmark as:

$$\Delta_t = \sum_{i=1}^{N} w_i R_{it} - R_{bt} = R_{pt} - R_{bt}.$$

Notice two things about the return differential Δ. First, given the returns to the N assets in the managed portfolio and the benchmark, it is a function of the investment weights that the manager selects (i.e., $\Delta = f(\{w_i\}/\{R_i\}, R_b)$). Second, Δ can be interpreted as the return to a hedge portfolio where $w_b = -1$.

With these definitions and a sample of T return observations, calculate the variance of Δ as follows:

$$\sigma_\Delta^2 = \frac{\sum_{t=1}^{T} (\Delta_t - \bar{\Delta})^2}{(T-1)}.$$

Then, the standard deviation of the return differential is:

$$\sigma_\Delta = \sqrt{\sigma_\Delta^2} = \text{periodic tracking error},$$

so that **annualized tracking error (TE)** can be calculated as:

$$TE = \sigma_\Delta \sqrt{P}$$

where P is the number of return periods in a year (e.g., P = 12 for monthly returns, P = 252 for daily returns).

TEV & ENHANCED INDEX FUND (similar to Core & Swing)

- Suppose, enhanced index fund = 10% active + 90% indexed
- Let tracking error of active = 5%
- Then, tracking error of enhanced index fund = 5%*10% = 0.5%

Subscripts: p = enhanced index portfolio; i = indexed portfolio; b = benchmark; a = active
Notation: r = return; w = weight; Var = variance; std = standard deviation; Corr = correlation

$r_p = w_i * r_i + w_a * r_a$

$r_p - r_b = w_i * (r_i - r_b) + w_a * (r_a - r_b)$, since $w_i + w_a = 1$.

Supposing $r_i = r_b$ then $r_p = w_a * (r_a - r_b)$

$Var(r_p - r_b) = Var\{w_i * (r_i - r_b)\} + Var\{w_a * (r_a - r_b)\} + 2 * w_i * w_a * Corr(r_i - r_b, r_a - r_b) * std(r_i - r_b) * std(r_a - r_b)$

$Var(r_p - r_b) = Var\{w_a * (r_a - r_b)\}$ since $r_i = r_b$

$std(r_p - r_b) = w_a * std(r_a - r_b)$

$$TEV = \omega\sqrt{\sigma^2_{Swing} + \sigma^2_{Bench} - 2Cov_{(Swing,\ Bench)}}$$

TEV RISES WITH BENCHMARK VOLATILITY

Subscript: p = portfolio; b = benchmark
Notation: r= return in excess of cash; e = error term; Var = variance;
β = beta in a single index market model

$r_p = \beta * r_b + e$

$r_p - r_b = (\beta - 1) * r_b + e$

$Var(r_p - r_b) = (\beta - 1)^2 * Var(r_b) + Var(e)$

There would be no correlation between r_b and the error term due to the regression.

TEV AND BETA

- Consider a combination of the market portfolio and cash.

- Subscript: m = market in the context of a single index market model; p = portfolio
- Notation: r = return in excess of cash; w = weight; β = beta; Var = variance;
- Cov = covariance; | | = absolute value

- rp = w*rm + (1-w)*0 = w*rm , since the excess return of cash is zero.

  β = Cov(rp , rm) / Var(rm) = w * Var(rm) / Var(rm) = w

- rp - rm = (w-1)*rm = (β-1)*rm

- Var (rp - rm) = (w-1)2 * Var(rm) = (β-1)2 * Var(rm)

- Tracking error = |w-1| * std(rm) = |β-1| * std(rm)