A stochastic optimization model for gas retail with temperature scenarios and oil prices parameters

F. Maggioni¹, M.T. Vespucci², E. Allevi³, M.I. Bertocchi⁴, R. Giacometti⁵ and M. Innorta⁶.

The paper deals with a new stochastic optimization model, named OMoGaS–2SV (Optimization Modelling for Gas Seller–Second Stochastic version), to assist companies dealing with gas retail commercialisation. We consider as source of stochasticity temperature, but we take into account also information on energetic indices. Temperature influences gas consumption of small consumers and is modelled by a mean reverting process. Oil prices and exchange rates influence the energetic indices to which sell and purchase prices are related. Forward curves of energetic indices are analyzed by a vector-autoregressive model while exchange rates are modelled by GARCH model. The profit function depends on the number of contracts with the final consumers, the typology of such consumers, the cost supported to meet the final demand and penalties for daily maximum consumption exceeding daily maximum capacity. Linear constraints related to a maximum daily gas consumption and binary constraints on indexation formulas are included.

Key Words. Gas sale company, energetic indices, mean reverting process, stochastic programming.
1 Introduction

Starting in 1999 the Italian Natural Gas market has been undergoing a liberalisation process aiming at promoting competition and efficiency, while ensuring adequate service quality standards. Timings and methods for the internal gas market liberalisation have been introduced following the European Gas Directive; the roles of different segments of the natural gas “chain” have been identified and defined, such as import, production, export, transportation and dispatching, storage, distribution and sale. In particular, the principle has been introduced of unbundling among supply and transport/storage and among distribution and selling. Before liberalization there was a national monopolistic operator, for all activities related to supply, transport, storage and wholesale commercialization, and local monopolistic operators, for distributing and selling to final consumers. After liberalization the following operators run different activities:

- shippers: production/import, re-gasification and wholesale commercialization;
- national distributor: transport on national network and storage;
- local distributors: transport on local network;
- selling companies: purchase gas from shippers and sell it to final consumers.

In 2003 the Italian Regulatory Authority for Electricity and Gas, see [7], defined consumption classes, on the basis of gas consumption in the thermal year, and introduced a new gas tariff with Delibera 134_06 in order to guarantee small consumers’ protection by applying the transparency principle in the pricing mechanism. The new tariff is based on a detailed splitting in different components, whose values are periodically revised, and represents a maximum price to be applied to small consumers. On the other hand, the gas buying price is influenced by some energetic indices related to the five following oil prices: Gasoil 0.2, 1% Fuel Oil, 3.5% Fuel Oil, Brent Dated and a mixture of Crude Oils, expressed in $/(Metric Ton) and whose daily quotations are available on Platts [12]. Consequently, in the purchase contract, the gas seller can choose among 17 possible energetic indices formulas given by different linear combinations of five oil prices reported above and by which the gas purchase price is monthly updated. Furthermore, unlike domestic customers, who purchase gas according to the index formula described by Delibera 134_06, industrial customers can choose the index formula by which the gas purchase price is computed, which implies that the gas seller faces the problem of develop a Risk Management Area for Oil Commodities to evaluate and control this market risk.

Two optimization models, a deterministic one [1] and a stochastic one [14], have been developed to assist companies dealing with retail commercialization. For each citygate, the gas seller has to decide the customer portfolio, i.e. the number of final customers to supply in each consumption class, and the sell prices to apply to each consumption class. Indeed, different customer portfolios determine different citygate consumption patterns, which shippers refer to when setting the gas price to be paid by the gas seller for the citygate supply. For each thermal year and each citygate there is a contract between shipper and gas seller setting:

- the gas volume required by gas seller in the next thermal year;
• the gas volume required in particular in winter months;
• the maximum daily consumption (capacity) requested by gas seller;
• the purchase price fixed by the shipper;
• energetic index formula.

In the contract it is also specified how to compute penalties, due by gas seller if daily consumption exceeds daily capacity.

The stochasticity considered in [14] is due to the influence of the temperature on consumption whereas sell and purchase prices do not change during the contract. For small customers, using gas either only for cooking or for cooking and heating, or for commercial activities and small industries, gas consumption in winter months strongly depends on the weather conditions: this fact is taken into account in the model, by including a mean reverting process modelling temperature, which gas consumption depends on and Monte Carlo scenarios simulation.

In this paper we consider again temperature as source of stochasticity but we also consider the influence of oil prices on the energetic indices upon which sell and purchase prices are monthly updated. We have modelled oil prices evolution by an endogenous VAR(5) econometric model with five lags, the exchange rates €/$ by an IGARCH model and finally we have built the forward curves of energetic indices by Monte Carlo scenarios simulation on the errors.

The models for temperature is presented in [14] while the model for energetic indices is presented in sections 2. In section 3 the stochastic model, named OMoGaS–2SV, is presented and in section 4 numerical results related to a case study are reported and discussed.

2 Forward curves of energetic indices

2.1 Econometric model of the forward curves of energetic indices

In this section we introduce the econometric model which describes oils evolution which the gas price depends from. The considered oils are Gasoil 0.2, a primary distillation of crude oil, 1% Fuel Oil and 3.5% Fuel Oil, respectively a low and high sulphur concentration fuel oils, Brent Dated, a crude oil of North Europe and a mixture of Crude Oils of Arabian countries. We have analyzed the database of these oils prices expressed in $/(Metric ton) from January 1998 to June 2005; their behaviours against time are plotted in Figure 1, in particular we have used monthly data because we are interested in long period forecasting.

From Figure 1 we can deduce some characteristics of these oils prices: positive correlation (see also the correlation matrix reported in Table 1), non-stationarity, non-trend-stationarity and an increasing trend in the last 6 observations. In order to test non-stationarity and non-trend-stationarity of the series we have used the unit-root test (see Dickey & Fuller (1979), [8]); the same test has been used to check the stationarity of returns of prices. The stationarity of errors of the regression between oil prices (we prefer
to make forecast on prices and not on their first differences) has been tested through cointegration (see Engle & Granger 1987, [9]) analysis: it can happen, in fact, that price series are non-stationary, but a linear combination of them is stationary. On this purpose we have checked the prices series cointegration by using the Johansen’s procedure (see [13]) based on the trace test.

<table>
<thead>
<tr>
<th>oils prices</th>
<th>Gasoil 0.2</th>
<th>1% Fuel O.</th>
<th>3.5% Fuel O.</th>
<th>Brent Dated</th>
<th>mix Crude O.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gasoil 0.2</td>
<td>1.0000</td>
<td>0.92527</td>
<td>0.89595</td>
<td>0.98551</td>
<td>0.97287</td>
</tr>
<tr>
<td>Fuel O.</td>
<td>0.92527</td>
<td>1.0000</td>
<td>0.97617</td>
<td>0.93915</td>
<td>0.90788</td>
</tr>
<tr>
<td>3.5% Fuel O.</td>
<td>0.89595</td>
<td>0.97617</td>
<td>1.0000</td>
<td>0.92699</td>
<td>0.88928</td>
</tr>
<tr>
<td>Brent Dated</td>
<td>0.98551</td>
<td>0.93915</td>
<td>0.92699</td>
<td>1.0000</td>
<td>0.9732</td>
</tr>
<tr>
<td>mix Crude O.</td>
<td>0.97287</td>
<td>0.90788</td>
<td>0.88928</td>
<td>0.9732</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Table 1: Correlation matrix of the oils prices.

The second step is to estimate the regression model. In order to reduce price volatility, we have considered the logarithm of prices. Moreover, in order to capture the evolution and the interdependencies between the five price time series, we have used a vectorial autoregressive model VAR(p):

\[
Y_t = c + A_1 Y_{t-1} + \cdots + A_p Y_{t-p} + \epsilon_t, \tag{1}
\]

where \(Y_t, \ldots, Y_{t-5}\) are 5 x 1 vectors, \(c\) is 5 x 1 vector of constants, \(A_i\) (for every \(i = 1, \ldots, p\)) are 5 x 5 matrices and \(\epsilon_t\) is a 5 x 1 vector of error terms with mean equal to zero \((E(\epsilon_t) = 0)\), variance–covariance matrix \(E(\epsilon_t \epsilon_t')\) given by a 5 x 5 positive definite matrix and without correlation across time \((E(\epsilon_t \epsilon_{t-k}) = 0, \forall k \neq 0)\). By using the Granger’s causality technique (see [9]), we have also tested that the VAR model is endogenous and finally, through some tests of correct specification on the errors and
checking the forecasting test reported below in Figure 2, we have concluded that the best model to describe the oil prices evolution is a VAR(5) with five lags given by eq. (1) with $p = 5$, see [15] for detailed results on coefficients.

### 2.2 Monte Carlo scenarios of the forward curves of energetic indices

Scenarios of (forecasted) oil prices have been generated by a Monte Carlo simulation. The error terms $\epsilon_i (i = 1, \ldots, 5)$ of the VAR(5) model (see eq. (1)) are correlated normally distributed random variables, thus they can be described by the Brownian motion

$$\epsilon_i^j := Y_i^j - Y^i, \quad dY_i^j = \mu_i dt + \sigma_i dW_i^j, \quad i = 1, \ldots, 5,$$

where $Y^i$ are the observed values, $\mu_i$ and $\sigma_i$ are respectively the mean and the variance of the errors series and $W_i^j$ is the Wiener process. In our particular case eq. (2) is given by:

$$\begin{align*}
\epsilon_1^1 &= -0.80242 dt + 19.862 dW_1^1, \\
\epsilon_2^1 &= -0.47046 dt + 12.42 dW_2^1, \\
\epsilon_3^1 &= -0.39153 dt + 10.399 dW_3^1, \\
\epsilon_4^1 &= -0.71279 dt + 16.968 dW_4^1, \\
\epsilon_5^1 &= -0.67033 dt + 17.797 dW_5^1,
\end{align*}$$

where we note that the Wiener processes $W_i^j$ are correlated; we have decomposed the correlation matrix $\Gamma$ of the errors (see Table 2), by using the *Cholesky decomposition* given by

$$\Gamma = C^T C,$$

Figure 2: Forecasting analysis of the endogenous model VAR(5) for the oils prices time series, with exclusion of the first 24 observations.
where $C$ is the lower triangular Cholesky matrix, reported in Table 3, from which eq. (2) can be rewritten as

$$
\epsilon_t^i = dY_t^i = \mu_t dt + \sigma_t \sum_{j=1}^5 c_{ij} d\tilde{W}_j^i, \quad i = 1, \ldots, 5,
$$

(4)

where $\tilde{W}_j^i$ are independent Wiener processes.

<table>
<thead>
<tr>
<th></th>
<th>Gasoil 0.2</th>
<th>1% Fuel O.</th>
<th>3.5% Fuel O.</th>
<th>Brent Dated</th>
<th>mix Crude O.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gasoil 0.2</td>
<td>1.0000</td>
<td>0.77755</td>
<td>0.7091</td>
<td>0.87108</td>
<td>0.8406</td>
</tr>
<tr>
<td>Fuel O.</td>
<td>0.77755</td>
<td>1.0000</td>
<td>0.8185</td>
<td>0.75532</td>
<td>0.81416</td>
</tr>
<tr>
<td>3.5% Fuel O.</td>
<td>0.7091</td>
<td>0.8185</td>
<td>1.0000</td>
<td>0.7581</td>
<td>0.83911</td>
</tr>
<tr>
<td>Brent Dated</td>
<td>0.87108</td>
<td>0.75532</td>
<td>0.7581</td>
<td>1.0000</td>
<td>0.93284</td>
</tr>
<tr>
<td>mix Crude O.</td>
<td>0.8406</td>
<td>0.81416</td>
<td>0.83911</td>
<td>0.93284</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Table 2: Correlation matrix of the errors $\epsilon_t^i i = 1, \ldots, 5$ of VAR(5) model.

<table>
<thead>
<tr>
<th></th>
<th>Gasoil 0.2</th>
<th>1% Fuel O.</th>
<th>3.5% Fuel O.</th>
<th>Brent Dated</th>
<th>mix Crude O.</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gasoil 0.2</td>
<td>1.0000</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Fuel O.</td>
<td>0.77755</td>
<td>0.62882</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>3.5% Fuel O.</td>
<td>0.7091</td>
<td>0.42483</td>
<td>0.56276</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Brent Dated</td>
<td>0.87108</td>
<td>0.12406</td>
<td>0.15586</td>
<td>0.44893</td>
<td>0</td>
</tr>
<tr>
<td>mix Crude O.</td>
<td>0.8406</td>
<td>0.25532</td>
<td>0.23913</td>
<td>0.29328</td>
<td>0.29155</td>
</tr>
</tbody>
</table>

Table 3: Choleski matrix $C$ obtained by the decomposition of the correlation matrix $\Gamma = C^T C$.

### 2.3 Forecasting analysis for exchange rates

In this section we consider the problem of simulating exchange rates between dollar and Euro; the oil prices are in fact expressed in dollar, while in the contracts between the shipper and gas retailer seller the price of gas is in Euro. We have analyzed the database of the exchange rates expressed in €/$ from January 1998 to June 2005, their monthly and daily behaviors against time; we have also analyzed their returns. The related plots are shown in Figure [15]. From unit–root test and the Bera–Jarque test on normality, we can deduce that the exchange rate returns series is stationary whereas the exchange rate series are not. Also, both are not normally distributed. In order to find a model which describe the behavior of exchange rates, we have used the hypothesis of market efficiency (i.e. most of the information is described by nearest observations): this means other economical variables need not to be included in the model. The first step in our analysis has been to estimate an autoregressive model AR($p$) varying the number of lags $p = 2, \ldots, 4$. We obtained that the best process for daily returns of exchange rates is an AR(2) whereas for the monthly case is an AR(1). In both cases the low values of $R^2$-test (see e.g. Davidson, (2000), [6]) bring us to consider this modelization unreliable.
We have modelled the returns of exchange rates \( r_t \) by a *generalized autoregressive conditional heteroskedasticity model* GARCH\((p,q)\) (see Bollerslev, (1986), [4]) as follows

\[
r_t = c + \epsilon_t ,
\]

where \( c \) is a constant, \( \epsilon_t = N(0, \sigma_t^2) = \sigma_t N(0, 1) \) is the error term such that its variance \( \sigma_t^2 \) at time \( t \) depends on the squared error terms from \( p \) previous periods and on \( q \) previous variances:

\[
\sigma^2_t = \alpha_0 + \sum_{j=1}^{p} \alpha_j \epsilon^2_{t-j} + \sum_{k=1}^{q} \beta_k \sigma^2_{t-k} .
\]

By using some tests of correct specification we found that the best model is an Integrated Generalized Autoregressive Conditional Heteroskedasticity IGARCH\((1,1)\),

\[
\begin{cases}
\sigma^2_t = \alpha_0 + \alpha_1 \epsilon^2_{t-1} + \beta_1 \sigma^2_{t-1} , \\
\alpha_1 + \beta_1 = 1 ,
\end{cases}
\]

which consists in a restricted version of the GARCH model, where the sum of the persistent parameters \( \alpha \) and \( \beta \) sum up to one. See [15] for coefficients obtained by our data.

Simulating exchange rates by a Monte Carlo method, we built gas prices scenarios, according to different indices formulas, denoted in the sequel by \( \psi = 1, \ldots , 17 \).

### 3 The stochastic OMoGaS-2SV model

In the literature [5], [10], [11] and [16] stochastic approaches in the gas market deals mainly with the scheduling of development of gas fields, the use of gas storage and the gas delivery problem. The stochasticity is introduced through a stochastic data process \( \omega = (\omega_1, \omega_2) \) where the first component \( \Delta \) represents the temperature and the second component \( I_\psi \) represents the oil index price along the year. The stochastic version of our model is built on scenarios of future consumptions, where consumptions of the first six classes depend on temperature variations along the year. We also consider the stochastic nature of purchase and sell prices monthly updated according to energetic indices related to oil prices.

The following notation is used:

- \( I = \{i = 1, \ldots , 12\} \) is the set of month indices, with \( i = 1 \) corresponding to July and \( i = 12 \) corresponding to the following June;
- \( J = \{j = 1, \ldots , 10\} \) is the set of consumer class indices;
- \( \Psi = \{\psi = 1, \ldots , 17\} \) is the set of energetic indices formulas;
- \( S = \{s = 1, \ldots , N\} \) is the set of scenario indices;
- \( c^s_{ij} \) is the consumption of consumer \( j \), \( 1 \leq j \leq 6 \), in month \( i \) along scenario \( s \)

\[
c^s_{ij} = \bar{C}_{ij} + C_{ij} \Delta^s_i ,
\]
where $\bar{C}_{ij}$ is the average consumption of consumer $j$ in month $i$; for $7 \leq j \leq 10$ the consumption does not depend on temperature and therefore
\[ c_{ij} = \bar{C}_{ij} ; \] (9)

- $va_j^s$ is the annual volume of gas for consumer $j$, $1 \leq j \leq 6$, in month $i$ along scenario $s$
\[ va_j^s = \sum_{i=1}^{12} c_{ij}^s ; \] (10)
for $7 \leq j \leq 10$ the annual volume of gas is
\[ va_j = \sum_{i=1}^{12} \bar{C}_{ij} ; \] (11)

- $vw_j^s$ is the winter volume of gas for consumer $j$, $1 \leq j \leq 6$, in month $i$ along scenario $s$
\[ vw_j^s = \sum_{i=5}^{9} c_{ij}^s ; \] (12)
for $7 \leq j \leq 10$ the winter volume of gas is
\[ vw_j = \sum_{i=5}^{9} \bar{C}_{ij} ; \] (13)

- $r_j^s$ is the ratio of winter gas consumption with respect to the total annual consumption of consumer $j$, $1 \leq j \leq 6$, in month $i$ along scenario $s$
\[ r_j^s = \frac{vw_j^s}{va_j^s} ; \] (14)
for $7 \leq j \leq 10$ the ratio of winter gas consumption with respect to the total annual consumption is
\[ r_j = \frac{vw_j}{va_j} ; \] (15)

- $cd_{ij}^s$ is the peak consumption per day of customer $j$ in month $i$ for $s = 1, \ldots, N$
\[ cd_{ij}^s = c_{ij}^s \frac{\gamma}{t_i} , \] (16)
where $t_i$ is the number of days of the month $i$ and $\gamma$ is a parameter given by the Authority;

- $nc_j$ are the first stage decision variables representing the number of consumers of class $j$, restricted to be nonnegative integers, subject to upper bounds, $\bar{nc}_j$,
\[ 0 \leq nc_j \leq \bar{nc}_j \quad j \in J ; \]
• $cm_i^s$ is the citygate consumption of month $i$ along scenario $s$

$$cm_i^s = \sum_{j=1}^{6} c_{ij}^s \cdot nc_j + \sum_{j=7}^{10} c_{ij} \cdot nc_j, \quad i \in I; \quad (17)$$

• $ca^s$ is the gas volume to be purchased for supplying the citygate consumers along scenario $s$

$$ca^s = \sum_{i=1}^{12} cm_i^s; \quad (18)$$

• $x^s$ is the citygate loading factor along scenario $s$ and $g$ is the first stage decision variable representing the maximum consumption per day above which the gas seller has to pay a penalty

$$x^s = \frac{ca^s}{365 \cdot g}; \quad (19)$$

• $l_j$ is the loading factor of consumer class $j$, $7 \leq j \leq 10$;

• $s_{ki}^+, k = 0, 1, 2$ are second stage decision variables along scenario $s$ that represent the surplus of consumption in the peak day of winter month $i$ ($i = 5, \ldots, 9$) with respect to gas availability given by the decision variable $g$. These variables are used in computing the penalties by $\sum_{i=5}^{9} \sum_{k=1}^{2} \mu_{ki} s_{ki}^+$ where $\mu_{ki}$ is the unitary penalty in month $i$ to be paid on the amount $s_{ki}^+$. The unitary penalty $\mu_{0i}$ is zero and the surplus variables $s_{ki}^+$ must satisfy the relations

$$0 \leq s_{0i}^+ \leq \pi_{0i} \cdot g, \quad i \in I, \quad s \in S,$$

$$\pi_{0i} \cdot g \leq s_{1i}^+ \leq \pi_{1i} \cdot g, \quad i \in I, \quad s \in S,$$

$$\pi_{2i} \cdot g \leq s_{2i}^+, \quad i \in I, \quad s \in S,$$

where $\pi_{ki}$ represents the width of penalizations classes $k = 0, 1$ (no upper bound for class $k = 2$);

• $cw^s$ is the citygate consumption in winter months along scenario $s$

$$cw^s = \sum_{i=5}^{9} cm_i^s; \quad (20)$$

• $h^s$ is the ratio of winter gas consumption with respect to total annual consumption along scenario $s$

$$h^s = \frac{cw^s}{ca^s}; \quad (21)$$

• $\phi_{\psi}$, ($\psi \in \Psi$) are first stage decision binary variables such that

$$\sum_{\psi=1}^{17} \phi_{\psi} = 1; \quad (22)$$
We choose as objective function the expected value of the gas seller profit:

\[ w = E[R(nc_j, g) - C(nc_j, g) - Pt(s_{ki}^{\psi}, g)] \]  

(26)
where
\[
R(nc_j, g) = \sum_{j=1}^{12} \sum_{i=1}^{10} P^s_{ij+} \cdot c^s_{ij} \cdot nc_j + \sum_{j=7}^{12} \sum_{i=1}^{17} \sum_{\psi_s=1}^{12} \phi_{\psi_s} P^s_{ij+} \cdot \bar{C}_{ij} \cdot nc_j ,
\]
represents the revenue,
\[
C(nc_j, g) = \sum_{i=1}^{12} \sum_{\psi_b=1}^{17} \phi_{\psi_b} P^s_{ij+} \cdot cm^s_{ij} ,
\]
represents the costs and
\[
Pt(s^+_k, g) = \sum_{i=5}^{2} \sum_{k=0}^{s^+_k} \mu_{ki} s^+_k .
\]
represents the penalties of gas retailer.

The constraints of our stochastic problem are the following:
\[
0 \leq nc_j \leq \bar{nc}_j \quad j \in J
\]
\[
\sum_{j=1}^{6} cd^g_{ij} \cdot nc_j + \sum_{j=7}^{10} cd_{ij} \cdot nc_j - g \leq \sum_{k=0}^{s^+_k} s^+_k \quad i \in I , \quad s \in S
\]
\[
0 \leq s^+_0 \leq \pi_0 \cdot g , \quad i \in I ,
\]
\[
\pi_0 \cdot g \leq s^+_1 \leq \pi_1 \cdot g , \quad i \in I ,
\]
\[
\pi_2 \cdot g \leq s^+_2 , \quad i \in I ,
\]
\[
\sum_{\psi_s=1}^{17} \phi_{\psi_s} = 1 , \quad \sum_{\psi_b=1}^{17} \phi_{\psi_b} = 1 .
\]

For details on the computation of the expected values see [15]. In [15] it is also shown that we take only the temperature as unique source of stochasticity because the indices formulas on oil prices appear only as expected value in the objective function and not in constraints; consequently we can drop oil prices as random variable and consider these as parameters which are monthly updated.

Notice that the problem may also be formulated as a 2-stage stochastic model with recourse as follow:
\[
\max E_{\xi} [f(x,y(\omega_1))] ,
\]
\[
Ax = b ,
\]
\[
T(\omega_1)x + W y(\omega_1) = h(\omega_1) ,
\]
\[
x \geq 0 , \quad y(\omega_1) \geq 0 ,
\]
where \(\xi = (h(\omega_1), T(\omega_1))\) is a random vector influenced by random temperature data. In our problem the first stage decision variables \(x\) involves:
• the number of customers $nc_j$ of class $j \in J$;
• the daily capacity $g$ above which the gas seller has to pay a penalty;
• binary variables $\phi_\psi, \psi \in \Psi$;

whereas the second stage decision variable $y(s)$ involves the surplus in consumption in the peak day $s_{ki}^+$ in winter month $i$. Furthermore the first stage constraint (37) are represented by equations (30) and (35) and the second stage constraint (38) by equation (31), (32), (33) and (34).

4 Results and model validations

In this section, we show the results of our stochastic model for a local gas seller who has to decide the customer portfolio structure in a village in Northern Italy (Sotto il Monte). The simulation is based on the data of thermal year 2004-2005 (for these data see [2]). We have developed a simulation framework based on ACCESS 97, for database management, on MATLAB release 12, for data visualization, and on GAMS release 21.5, for optimization. In the GAMS framework the DICOPT solver has been used for the nonlinear mixed integer optimization problem. DICOPT solves a series of NLP subproblems by CONOPT2 and MIP subproblems by CPLEX.

The relation between the part of the purchase price $P^*$, which does not depend by oil prices index formulas, and $x^s$ is estimated by the gas seller through a linear regression using the data related to year 2004-2005 for all citygates managed by the gas seller. The regression of $P^*$ values has also been tried on the annual volume $ca_s, h^s$ and $g$ but it has been found not significant. Indeed, the value of $R^2$-test (see e.g. Davidson, (2000), [6]) with the regression on $x^s$ is 0.603, therefore not highly significant. However, the introduction of non parametric regression, would introduce a more complicated function in the model. On the other side, linear regression is currently used by the gas seller in their simulations. In our case we use:

$$P^*(x^s) = QT + QS + 18.348 - 3.866 \cdot x^s,$$

(40)

where the intercept value 18.348 and the slope value $-3.866$; the values $QT$ and $QS$ are given by the Italian Regulatory Authority: in our numerical experiments $QT = 2.4953171$ Eurocent/Stm$^3$ and $QS = 0.63882$ Eurocent/Stm$^3$.

The relation between the consumption of consumer $j$, $1 \leq j \leq 6$, in month $i$ along scenario $s$, $c^s_{ij}$ and the deviation from mean value over scenarios, $\Delta^s_{ij}$, is supposed to be linear with intercept equal to $\bar{C}_{ij}$ and the other coefficient computed via a linear regression. The regression results to be significative for all the consumers.

In particular, we choose to work under the assumption that all the contracts on sale and on purchase are stipulated in the same month (i.e. $i^* = 1$), obviously we can modify the objective function to take into account of possible diversified effective dates.

The model has been validated by running several tests both in the deterministic (see [1]) and in the stochastic case. For the stochastic model, we have solved 10000 times the problem, each time with $N = 80$ scenarios randomly choosen with the procedure described in [14]. The optimal values both in the function and in the decision variables
are stable. We report in Table 4 their average over 10000 trials. The first column refers to the solution obtain by a first stochastic model described in a previous paper (see [14]) in which the source of stochasticity was again given by the dependence of consumption by the temperature but the purchase and sell prices did not change during the year, whereas the second column refers to the model described here in which is also taken into account the dependence of gas prices by energetic indices; in Table 5 are also reported the optimal values of purchase prices along the months in a year obtained as solutions of second stochastic model.

By looking Table 4 we can note that, even if the two models are different and consequently the correspondent results not comparable, the optimal profit value of the second model, is much higher than the one of the first model; this is due to the fact that gas retailer can choose to buy according to the lowest oil prices index formula $\psi_b$ and sells to industrial customers at highest one $\psi_s$.

In Figure 3 are reported the forward mean values along 1000 simulations of the energetic indices $I_\psi - I_\psi^0$, with $\psi \in \Psi$, in the period between July 2005 and June 2006. As expected, the optimal purchase index formula is given by $\psi_b = 11$ which corresponds to the lowest index formula 1% Fuel Oil 12,1,1, and the optimal sell formula is $\psi_s = 13$, that is the highest index formula, C 6,1,1.

Another test has also been considered with the aim to quantify the losses in the gas seller budget in the case of non optimal purchase and sell index formulas. In this case we have fixed $\psi_b = \psi_s = 1\% 3.5\%$ Fuel Oil 9,1,1 (see Figure 3) and we have reported in the first column of Table 6 the relative optimal profit value compared to the case of optimal index formulas $\psi_b = 11$ and $\psi_s = 13$.

<table>
<thead>
<tr>
<th>First stochastic</th>
<th>Second stochastic</th>
<th>Euro</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profit</td>
<td>151713</td>
<td>197264</td>
</tr>
<tr>
<td>$P'$</td>
<td>19.67</td>
<td>23.85</td>
</tr>
<tr>
<td>$P''$</td>
<td>25.00</td>
<td>29.37</td>
</tr>
<tr>
<td>$P''$</td>
<td>19.88</td>
<td>26.52</td>
</tr>
<tr>
<td>$ca$</td>
<td>4484725</td>
<td>4484406</td>
</tr>
<tr>
<td>$g$</td>
<td>26319</td>
<td>26251</td>
</tr>
<tr>
<td>$x$</td>
<td>0.4669</td>
<td>0.4678</td>
</tr>
<tr>
<td>$\psi_b$</td>
<td>$\psi_b = 11$: 1% Fuel Oil 12,1,1</td>
<td>$\psi_s = 13$: C 6,1,1</td>
</tr>
</tbody>
</table>

Table 4: Optimal values for citygate Sotto il Monte respectively in the first (see [14]) and second stochastic case.

In [15] we have also considered a different sampling strategy which gives similar optimal values to those reported in Table 4. In [15] simulations are reported on the sensitivity of the optimal value function and optimal decision variables upon increasing the dimension of the sampling strategy.

Finally, in order to relate clients number to the best selling price on the market $\bar{P}'$ if small customers, or $\bar{P}''$ if industrial ones, we have also thought to introduce in the model the following non linear constraints

$$(\bar{P}'_j - P'_{ij(\psi_s=1)})\vartheta_j + (\bar{P}'_j - P'_{ij(\psi_s=1)} - m)(1 - \vartheta_j) \geq 0 \quad j=1,\ldots,6, \; i \in I, \; s \in S$$
Table 5: Optimal values of purchase prices in the different months in a year as solutions of second stochastic model.

<table>
<thead>
<tr>
<th>Month</th>
<th>$P_{b}(\psi_b=1%)$ Eurocent/Stm$^3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>January</td>
<td>20.80</td>
</tr>
<tr>
<td>February</td>
<td>21.39</td>
</tr>
<tr>
<td>March</td>
<td>22.01</td>
</tr>
<tr>
<td>April</td>
<td>22.63</td>
</tr>
<tr>
<td>May</td>
<td>23.10</td>
</tr>
<tr>
<td>June</td>
<td>23.69</td>
</tr>
<tr>
<td>July</td>
<td>24.36</td>
</tr>
<tr>
<td>August</td>
<td>24.92</td>
</tr>
<tr>
<td>September</td>
<td>25.43</td>
</tr>
<tr>
<td>October</td>
<td>25.77</td>
</tr>
<tr>
<td>November</td>
<td>25.96</td>
</tr>
<tr>
<td>December</td>
<td>26.21</td>
</tr>
</tbody>
</table>

Table 6: Optimal values for citygate Sotto il Monte respectively in the case of choose of non-optimal indices formula $\psi_b = \psi_s = 1\%$ 3.5% Fuel Oil 9,1,1 C 6,1,1 and second stochastic case.

<table>
<thead>
<tr>
<th>$Profit$</th>
<th>135561</th>
<th>197264</th>
<th>Euro</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P$</td>
<td>24.96</td>
<td>23.86</td>
<td>Eurocent/Stm$^3$</td>
</tr>
<tr>
<td>$P'$</td>
<td>29.38</td>
<td>29.38</td>
<td>Eurocent/Stm$^3$</td>
</tr>
<tr>
<td>$P^{''}$</td>
<td>25.17</td>
<td>26.53</td>
<td>Eurocent/Stm$^3$</td>
</tr>
<tr>
<td>$ca$</td>
<td>4484656</td>
<td>4484406</td>
<td>Stm$^3$</td>
</tr>
<tr>
<td>$g$</td>
<td>26264</td>
<td>26251</td>
<td>Stm$^3$</td>
</tr>
<tr>
<td>$x$</td>
<td>0.4678</td>
<td>0.4678</td>
<td></td>
</tr>
<tr>
<td>$\psi_b$</td>
<td>1% 3.5% Fuel Oil 9,1,1</td>
<td>1% Fuel Oil 12,1,1</td>
<td></td>
</tr>
<tr>
<td>$\psi_s$</td>
<td>1% 3.5% Fuel Oil 9,1,1</td>
<td>C 6,1,1</td>
<td></td>
</tr>
</tbody>
</table>

\[
(\bar{P}^{''}_j - P^{''}_{ij\psi_s})\theta_j + (\bar{P}^{''}_j - P^{''}_{ij\psi_s} - m)(1 - \theta_j) \geq 0 \quad j=7,\ldots,10, \ i \in I, \ s \in S
\]

\[nc_j \leq \theta_j \bar{n}c_j \quad j \in J ,\]

where $\theta_j$ ($j \in J$) is a binary variable, $\bar{n}c_j$ is the maximal number of customer class $j \in J$ and $m$ a negative number such that $|m| \gg (\bar{P}_j - P_{ij})$, $i \in I, j \in J$. By the way we have not taken into account in the model these constraints because they add a further computational complexity which is not solvable by standard algorithm.

5 Conclusions

We have proposed a stochastic model for the management of a gas retail company where we have considered temperature as source of stochasticity, but we have taken into account also information on oil prices to which gas purchase and sell prices are related and monthly updated. In order to allow the gas seller to evaluate the possibility of a client to migrate to
other providers, a possible extension of the model consists in including linear constraints among prices and number of clients. Finally, there are various possible extensions of the model, for instance including different type of contracts with the shipper (portfolio management of gas contracts) or considering storage facilities for a gas shipper-seller.

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References


