Dynamic Asset-Liability management and decision support for Pension Funds

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Outline

1. PF economics
2. Stochastic optimization model
3. Risk exposure
4. DSP interfaces and implementation
5. Case study and numerical evidences
6. Conclusions
We focus on a PF portfolio management problem formulated as a dynamic stochastic program over a 20 year horizon with annual pension payments.

The PF manager is assumed to seek an optimal trade-off between short, medium and long-term goals. An extended asset universe is considered including liquid and illiquid assets.

The Pension Fund liability or *DBO* is computed as discounted value of inflation adjusted pension payments through an appropriate replicating portfolio of discount bonds and caps and floors.

We assume constant contribution rates and employee-sponsors sharing rule during the accumulation phase.
PF key variables

Net pension payments

Portfolio income return (coupon/dividends)

DBO

AlM Risk

Liquidity gap plus AlM Risk

Fair value on plan assets

NET DBO

Plan sponsors’ unexpected contributions (a portion of NetDBO)

Total Portfolio Return (coupon/dividends, capital gains and UGL)

Investment Risk Capital (AlM Risk and correlations)

Return on Plan Assets per unit tail risk
PF net pension payment

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ALM 4PF via dynamic stochastic optimization
The PF liability - specifically its Defined Benefit Obligation (DBO) - is computed as discounted values of inflation adjusted pension payments. The interest rate dynamics are assumed to be consistent with Black’s pricing model for interest-rate sensitive instruments.

\[ \sum_{T=t+1}^{M} \left( ZCB(t, T) - \text{CAPLET}(t, T) + \text{FLOORLET}(t, T) \right) \]
Liability pricing model: numerical example

We show that such portfolio actually matches the value of future nominal liabilities discounted at the real interest rates, computed by subtracting inflation caps or floors from the prevailing nominal interest rates.

Consider the liability cash flows expected at the end of years 1 and 2, evaluated at $t = 0$, with compounded inflation for payments to occur at $T$. In $t = 0$ we have:

<table>
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<tr>
<th>Year</th>
<th>Infl</th>
<th>ZCB</th>
<th>Cap</th>
<th>Floor</th>
<th>Rpl. ptf value</th>
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<td>T=1</td>
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<td>4,75</td>
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$$\text{DBO} = 302,699748$$
$$\text{Disc. value} = 303,15061$$
Liability pricing model: volatility estimation

Volatility fitting procedure

As for caps and floors, starting with two different matrices of option premiums (one for cap-contract prices and one for floor-contract prices), an optimal volatility surface for different tenors and strikes is computed that minimize the distance between the input data and the replicating portfolio.
For $t \in \mathcal{T}$, a discrete and finite time space, $n \in \mathcal{N}_t$ are nodal labels with children $n+$ and parent node $n−$. We denote by $a(n)$ the sequence of all ancestors for any final or intermediate node and with $c(n)$, the subtree originating from $n$. $t_n$ is the time of node $n$.

Variables at the intermediate stages:
- Income returns on fixed income assets,
- Expiring fixed income securities,
- Annual pension payments,
- Cash imbalances.

Variables at the decision stages:
- Income returns, capital gains, UGL on the extended asset universe,
- Expiring fixed income securities, Corporate sponsors’ contributions,
- Net pension payments, Interests on cash imbalances,
- Selling decisions and new investments.
Notation and model variables

For $t \in T$, $n \in \mathcal{N}_t$

- $\Omega_n$ liquidity gap: *net pension payments minus portfolio income return*;
- $K^1_n$ ALM Risk: *interest rate fluctuations times the duration gap*;
- $\Lambda_n$ defined benefit obligation (DBO): *present value of the accrued pension obligation*;
- $A_{t,s}$ fair value of plan assets: *change in fair value of plan assets determined by the total portfolio return*;
- $K^f_n$ Investment Risk Capital;
- $\Pi_{n}^{cum}$ total cumulated realized portfolio return: *unexpected additional contributions motivated by increasing funding gap*. 

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ALM 4PF via dynamic stochastic optimization
Notation and model variables

For $t \in T, n \in \mathcal{N}_t$

- $L_{n}^{NET}$ net pension payments
- $\Pi_{n}^{1,INV}$ portfolio income return
- $x_{i,m,n}$ are portfolio holdings of asset $i$, node $n$, bought in $m \in a(n)$. $X_{n}^{-} = \sum_{i} \sum_{m} x_{i,m,n}^{-}$ and $X_{n}^{+} = \sum_{i} x_{i,n}^{+}$ are sellings and buyings. $X_{i,n} = \sum_{m \in a(n)} x_{i,m,n}$ and $X_{n} = \sum_{i} X_{i,n}$.

For $t \in T_{int}, n \in \mathcal{N}_t$

- $L_{n}^{Z}$ cash imbalances over intermediate stages

\[
L_{n}^{Z} = X_{i,n}^{-} \cdot \xi_{i,n} + X_{i,n}^{exp} - L_{t,j,s}^{NET}
\]
For $t \in \mathcal{T}$, $n \in \mathcal{N}_t$

- Liquidity gap plus ALM risk: $Y_{1,n} = \Omega_n + K^1_n + \Psi_n$
- Plan sponsors’ unexpected contributions: $Y_{2,n} < \gamma \cdot \Phi^u_n$ with $\Phi^u_n := \Phi^u_{n,+} - \Phi^u_{n,-} = \Lambda_n - A_n$ where:
  - $\gamma$ plan sponsor contribution tolerance;
  - $\Phi^u_{n,+}, \Phi^u_{n,-}$ positive and negative part of $\Phi^u_n = Y_{2,n}$
- Return on plan assets per unit tail risk: $Y_{3,n} = \frac{\Pi^{cum}_n}{K^f_n}$
- Net defined benefit obligation (DBO): $Y_{4,n} = \Lambda_n - A_n$

The four targets are assumed to be set at the 1, 3, 10, 20 year horizon respectively.
The optimization problem is represented by:

\[
\max_{x \in X} \left[ (1 - \alpha) \cdot \sum_j \lambda_j E(Y_n|F_j) - \alpha \cdot \sum_j \lambda_j E(\tilde{Y}_j - Y_n|F_j) \right]
\]

s.t for all \( n \in \mathcal{N}_t, \ t \in \mathcal{T} \)

\[
X_n = X_n^-(1 + \rho_n) + X_n^+ - X_n^-
\]

\[
z_n = X_n^- - X_n^+ + \Phi_n^u + \Phi_n^d + \Pi_n^{1,INV} - L_n^{NET} + L_n^z(1 + \pi_n) + z_n^- (1 + r_n)
\]

\[
\Pi_n^f = X_n^- \xi_n + \sum_{m \in a(n)} X_{m,n}^- \gamma_{m,n} + \Pi_{n-}^f
\]

\[
X_n^+ + X_n^- = \vartheta X_n^- (1 + \rho_n)
\]

\[
K_n^f \leq \mu (X_n - \Lambda_n - K_n^t)
\]
Asset return model

The asset universe includes a set of investment opportunities, whose price $\rho$ and cash returns $\xi$ must be specified and input to the optimization problem:

- Barcap Treasury 1-5, 5-10, 10-20, 20-30 and 30+ year maturity
- 10 year Securitised, IG and SG Corporate Bond indices
- 10, 20 and 20+ year inflation-linked fixed income indices
- MSCI EMU (Public) Equity index and Private equity index
- Indirect GPR Europe real estate index
- Infrastructure Cyclical and Defensive investments (100 years)
- Renewable energy and Floaters
Investment risk capital

The risk capital consumption is given by:

\[ K_n^f = K_n^{f1} + K_n^{f2} \]  
\[ K_n^{f1} = (\Delta_{n-}^A - \Delta_{n-}^\Lambda) \, dr_n(t_n - t_{n-}) + K_{n-}^{f1} \]  
\[ K_n^{f2} = \sqrt{\sum_{i \in A} \sum_{j \in A} X_{i,n-} X_{j,n-} k_{ij} (t_n - t_{n-}) + K_{n-}^{f2}} \]

where \( \Delta_{n-}^A = \sum_i \sum_{m \in a(n)} x_{i,m,n} \Delta_{i,n} \) and \( \Delta_{n-}^\Lambda = \sum_{T > t_n} \Lambda_{n,T} \times (t - t_n) \) are the asset and liability durations in node \( n \). \( K_n^{f1} \) and \( K_n^{f2} \) define the interest rate and market risk exposure of the portfolio.
Implementation framework

System structure

Financial datafeed

Portfolio universe

Benchmark portfolio

Risk factors

Financial stochastic model

Monte Carlo simulator

Risk horizon

P&L distribution

Risk measures

Scenario generator

Dynamic portfolio model

SP generation

Stochastic program solution

Solution analysis

Gams 21.5

Matlab 7.4.0
Implementation framework

- Stage Setting
- Time Series
- Risk Factors
- Asset Universe
- Liabilities

Excel

Asset Return Model

Scenario Generation

Liability Model

MATLAB

results

GAMS

PF Asset-Liability Management Model
Implementation framework

Financial Series Sources

- Financial DataFEED
- DB manager
  - Investment Universe List
  - Financial series user-selected

Uncertainty model

- Risk factor Statistical Model (B1)
- MC simulator (B3)
- Event tree generator (B4)
- Scenario tree

Output validation

- Optimal Solution, EVPI, Benchmark analysis, Backtesting analysis

User-interface

- USER Interface (D1)
- Investment Policy Statement

Optimal choice model

- Optimal Results
- Optimization tool (C3)

- Dynamic Portfolio model (C1)
- DSP generator (C2)
Case study

We present a set of results focusing on:

- the liability pension payment module based on an underlying interest rate process consistent with the lognormal assumptions at the grounds of Black’s pricing model for interest-rate sensitive instruments;
- the generation and solution time for stochastic programs of increasing dimension under the different solution approaches;
- the scenario analysis of the obtained optimal solution.
The benefit indexation scheme is described by a replicating portfolio including a cap and a floor set of rates: the inflation adjustment pensions are revalued according to the occurred inflation but with a maximum possible revaluation and also with a protection of minimum revaluation.
Dynamic Global PF Asset Allocation

- **Actual Alloc. (dic 2013)**
  - Real Estate: 16%
  - Alternatives: 40%
  - Public Equity: 24%
  - Corporates: 24%
  - Securitized: 4%
  - TIPS: 4%
  - Treasuries: 14%
  - Cash: 16%

  - Real Estate: 16%
  - Alternatives: 16%
  - Public Equity: 24%
  - Corporates: 4%
  - Securitized: 39%
  - TIPS: 4%
  - Treasuries: 14%
  - Cash: 16%

**Dynamic Asset Allocation (million)**

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<td>77.506</td>
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<td>20v</td>
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- **Liquidity gap plus ALM risk:**
  - 492.563

- **Plan Sponsors’ Unexpected Contributions**
  - 441.855

- **Total Portfolio return per unit tail of risk**
  - -5.55%

- **Net Defined Benefit Obligation**
  - -317.061

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ALM 4PF via dynamic stochastic optimization
Scenario Analysis

Scenario Analysis Comments

1) Average in the Shortfall Distribution (Maximum in the Shortfall Distribution)

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<td>(123.241)</td>
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2) Total Portfolio return per unit tail of risk

3) Liquidity gap plus ALM risk

4) Net DBO

5) Plan Sponsors' Unexpected Contributions

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ALM 4PF via dynamic stochastic optimization
Investment risk capital exposure

Investment Risk Capital Bounds

- Investment Risk Capital (corr=1)
- Investment Risk Capital (diversified)
- Investment Risk Capital (corr=1)
- MaxRiskCap_{tol} (\equiv \mu * K_{max})
Surplus

Net DBO, Surplus (left axis) and Funding Ratio (right axis) (Mean)

- Deficit
- Surplus
- Net Defined Benefit Obligation
- Funding Ratio
Conclusions

- Pension Funds ALM is becoming an extremely relevant R&D area also in relatively conservative pension systems characterized by a majority of state-controlled tier 1 frameworks,

- We have analyzed the key elements of a DB ALM system developed for a large corporation, where a very simple liability model has been adopted,

- In a discrete model, large scale stochastic optimization provides an efficient methodology to tackle in a very realistic way several regulatory and financial issues affecting Pension Funds now-a-day.